Table A1: Enthalpy contribution expressions for M and R magmas

$h^{M}_{L\alpha}$	$f_o\left[C_S T^{M}_{o} + \Delta h^{\alpha} + X^{M}_{o}(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^{M}_{o}(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + (T^{M}_{o} - T^{\alpha}_{m.p.})\right)\right]$
$h^{M}_{L\beta}$	$f_o\left[C_S T^{M}_{o} + \Delta h^{\beta} + Y^{M}_{o}(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^{M}_{o}(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + (T^{M}_{o} - T^{\beta}_{m.p.})\right)\right]$
$h^{M}_{\alpha+L}$	$f_o\left[C_S T^{M}_{o} + \left(\frac{X^{M}_{o}}{X^{M\ell}_{o}}\right)\Delta h^{\alpha} + X^{M}_{o}(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^{M}_{o}(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + \left(\frac{X^{M}_{o}}{X^{M\ell}_{o}}\right)(T^{M}_{o} - T^{\alpha}_{m.p.})\right)\right]$
$h^{M}_{\beta+L}$	$f_o\left[C_S T^{M}_{o} + \left(\frac{Y^{M}_{o}}{Y^{M\ell}_{o}}\right)\Delta h^{\beta} + Y^{M}_{o}(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^{M}_{o}(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + \left(\frac{Y^{M}_{o}}{Y^{M\ell}_{o}}\right)(T^{M}_{o} - T^{\beta}_{m.p.})\right)\right]$
$h^{M}_{\alpha+\beta}$	$f_o\left[C_S T^{M}_{o}\right]$
$h^{R}_{L\alpha}$	$(1-f_o)\left[C_S T^{R}_{o} + \Delta h^{\alpha} + X^{R}_{o}(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^{R}_{o}(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + (T^{R}_{o} - T^{\alpha}_{m.p.})\right)\right]$
$h^{R}_{L\beta}$	$(1-f_o)\left[C_S T^{R}_{o} + \Delta h^{\beta} + Y^{R}_{o}(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^{R}_{o}(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + (T^{R}_{o} - T^{\beta}_{m.p.})\right)\right]$
$h^{R}_{\alpha+L}$	$(1-f_o)\left[C_S T^{R}_{o} + \left(\frac{X^{R}_{o}}{X^{R\ell}_{o}}\right)\Delta h^{\alpha} + X^{R}_{o}(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^{R}_{o}(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + \left(\frac{X^{R}_{o}}{X^{R\ell}_{o}}\right)(T^{R}_{o} - T^{\alpha}_{m.p.})\right)\right]$
$h^{R}_{\beta+L}$	$+(1-f_o)\left[C_S T^{R}_{o} + \left(\frac{Y^{R}_{o}}{Y^{R\ell}_{o}}\right)\Delta h^{\beta} + Y^{R}_{o}(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^{R}_{o}(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + \left(\frac{Y^{R}_{o}}{Y^{R\ell}_{o}}\right)(T^{R}_{o} - T^{\beta}_{m.p.})\right)\right]$
$h^{R}_{\alpha+\beta}$	$(1-f_o)\left[C_s\, T^{R}_{o}\right]$

Table A2: Specific enthalpy boundary values separating phase assemblages

Specific enthalpy	Fields Separated	Expressions for Specific enthalpy for $X^H<X_e$ and $X^H>X_e$
h_{max}	L and α+L	$C_S(T_e - T^{\alpha}_{m.p.})\left(\frac{X^H}{X_e}\right) + C_S T^{\alpha}_{m.p.} + \Delta h^{\alpha} + X^H(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^H(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + \frac{X^H}{X_e}(T_e - T^{\alpha}_{m.p.})\right)$
	L and β+L	$C_S(T_e - T^{\beta}_{m.p.})\left(\frac{Y^H}{Y_e}\right) + C_S T^{\beta}_{m.p.} + \Delta h^{\beta} + Y^H(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^H(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + \frac{Y^H}{Y_e}(T_e - T^{\beta}_{m.p.})\right)$
h_{mid}	α+L and L_e+α+β	$C_S T_e + \left(\frac{X^H}{X_e}\right)\Delta h^{\alpha} + X^H(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^H(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + \left(\frac{X^H}{X_e}\right)(T_e - T^{\alpha}_{m.p.})\right)$
	β+L and L_e+α+β	$C_S T_e + \left(\frac{Y^H}{Y_e}\right)\Delta h^{\beta} + Y^H(\Delta h^{\alpha} - \Delta h^{\beta}) + \Delta C\left(Y^H(T^{\beta}_{m.p.} - T^{\alpha}_{m.p.}) + \left(\frac{Y^H}{Y_e}\right)(T_e - T^{\beta}_{m.p.})\right)$
h_{min}	L_e+α+β and α+β	$C_S T_e$

h_{max}	L and $\alpha+\beta$	$C_S T_e + \Delta h^{\alpha} + X_e(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X_e(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + (T_e - T^{\alpha}_{m.p.})\right)$

Table A3: Hybrid magma state for $X^H < X_e$

Specific enthalpy range and phase assemblage	**Hybrid system state**
$h^H > h_{max}$ **L**	$T^H = \dfrac{\Phi h_o - \Delta h^{\alpha} - X^H(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(T^{\alpha}_{m.p.} - X^H(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.})\right)}{C_S + \Delta C}$ $X^{H\ell} = X^H$
$h_{mid} < h^H < h_{max}$ L+α	Simultaneous solution of the following two expressions gives $X^{H\ell}$ and T^H : $C_S T^H + \left(\dfrac{X^H}{X^{H\ell}}\right)\Delta h^{\alpha} + X^H(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X^H(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + \left(\dfrac{X^H}{X^{H\ell}}\right)(T^H - T^{\alpha}_{m.p.})\right) - \Phi h_o = 0$ $T^H = (T_e - T^{\alpha}_{m.p.})\dfrac{X^{H\ell}}{X_e} + T^{\alpha}_{m.p.}$ Mass fraction α crystals: $w^H_{\alpha} = 1 - \dfrac{X^H}{X^{H\ell}}$ Mass fraction melt: $w^H_{\ell} = \dfrac{X^H}{X^{H\ell}}$
$h_{min} < h^H < h_{mid}$ L_e+α+β	$T^H = T_e$ $X^{H\ell} = X_e$ mass fraction of liquid of eutectic composition: $w^H_{\ell} = \dfrac{\Phi h_o - C_S T_e}{\Delta h^{\alpha} + X_e(\Delta h^{\beta} - \Delta h^{\alpha}) + \Delta C\left(X_e(T^{\alpha}_{m.p.} - T^{\beta}_{m.p.}) + (T_e - T^{\alpha}_{m.p.})\right)}$ Mass fraction β phase: $w^H_{\beta} = X^H - w^H_{\ell} X_e$

	Mass fraction α phase: $w_\alpha^H = 1 - w_\beta^H - w_\ell^H$
$h^H < h_{min}$ $\alpha+\beta$	$T^H = \frac{\Phi h_o}{C_S}$ $w_\alpha^H = (1 - X^H)$ $w_\beta^H = X^H$

Table A4: Hybrid magma state for $X^H > X_e$

Specific enthalpy range	Hybrid system state
$h^H > h_{max}$	$T^H = \dfrac{\Phi h_o - \Delta h^\beta - Y^H(\Delta h^\alpha - \Delta h^\beta) + \Delta C\left(T^\beta_{m.p.} - Y^H(T^\beta_{m.p.} - T^\alpha_{m.p.})\right)}{C_S + \Delta C}$ $Y^{H\ell} = Y^H$
$h_{mid} < h^H$ h_{max}	Simultaneous solution of the following expressions gives melt composition $Y^{H\ell}$ and T^H : $C_S T^H + \left(\dfrac{Y^H}{Y^{H\ell}}\right)\Delta h^\beta + Y^H(\Delta h^\alpha - \Delta h^\beta) + \Delta C\left(Y^H(T^\beta_{m.p.} - T^\alpha_{m.p.}) + \left(\dfrac{Y^H}{Y^{H\ell}}\right)(T^H - T^\beta_{m.p.})\right) - \Phi h_o = 0$ $T^H = (T_e - T^\beta_{m.p.})\dfrac{Y^{H\ell}}{Y_e} + T^\beta_{m.p.}$ Mass fraction β crystals: $w^H_\beta = 1 - \dfrac{Y^H}{Y^{H\ell}}$ Mass fraction liquid: $w^H_\ell = \dfrac{Y^H}{Y^{H\ell}}$
$h_{min} < h^H < h_{mid}$	$T^H = T_e$ $Y^{H\ell} = Y_e$ mass fraction of liquid of eutectic composition: $w^H_\ell = \dfrac{\Phi h_o - C_S T_e}{\Delta h^\beta + Y_e(\Delta h^\alpha - \Delta h^\beta) + \Delta C\left(Y_e(T^\beta_{m.p.} - T^\alpha_{m.p.}) + (T_e - T^\beta_{m.p.})\right)}$ Mass fraction α phase: $w^H_\alpha = Y^H - w^H_\ell Y_e$ Mass fraction β phase: $w^H_\beta = 1 - w^H_\ell - w^H_\alpha$

$h^H < h_{min}$	$T^H = \frac{\Phi h_o}{C_S}$ $w_\alpha^H = Y^H$ $w_\beta^H = 1 - Y^H$

Table A5: Hybrid magma state for $X^H = X_e$

Specific enthalpy range	Hybrid system state
$h^H > h_{max}$	$T^H = \frac{\Phi h_o - \Delta h^\alpha - X_e(\Delta h^\beta - \Delta h^\alpha) - \Delta C\left(T_e - T^\alpha_{m.p.} + X_e(T^\alpha_{m.p.} - T^\beta_{m.p.})\right)}{C_S}$ $X^{H\ell} = X_e$
$h_{min} < h^H < h_{max}$	$T^H = T_e$ $X^{H\ell} = X_e$ Mass fraction of liquid of eutectic composition $w_\ell^H = \frac{\Phi h_o - C_S T_e}{\Delta h^\alpha + X_e(\Delta h^\beta - \Delta h^\alpha) + \Delta C\left(X_e(T^\alpha_{m.p.} - T^\beta_{m.p.}) + (T_e - T^\alpha_{m.p.})\right)}$ Mass fraction α phase: $w_\alpha^H = (1 - w_\ell^H)(1 - X_e)$ Mass fraction β phase: $w_\beta^H = (1 - w_\ell^H)X_e$
$h^H < h_{min}$	$T^H = \frac{\Phi h_o}{C_S}$ $w_\alpha^H = 1 - X_e$ $w_\beta^H = X_e$

Table A6: Thermodynamic parameters of toy model. Parameters closely follow those in system $CaMgSi_2O_6$-$CaAl_2Si_2O_8$ at 10^5 Pa.

Thermodynamic parameter	Symbol	Value	Units
Eutectic composition, mass fraction component B	X_e	0.42	
Eutectic temperature	T_e	1547	K
Melting point of α crystals	$T^{\alpha}_{m.p.}$	1665	K
Enthalpy of fusion of α crystals at $T^{\alpha}_{m.p.}$	Δh^{α}	636	kJ/kg
Melting point of β crystals	$T^{\beta}_{m.p.}$	1830	K
Enthalpy of fusion of β crystals at $T^{\beta}_{m.p.}$	Δh^{β}	478	kJ/kg
Crystal specific isobaric heat capacity	C_S	1400	J/kg K
Liquid specific isobaric heat capacity	C_L	1600	J/kg K