

Appendix I: Derivation of the Toy Model

Thermodynamic Parameters

The toy model is based upon the thermodynamics of an isobaric two-component (A and B) phase diagram. The definition of quantities is given in Table 1 in the text. Figure 1 gives the toy model phase diagram. X is the mass fraction of component B and Y is the mass fraction of component A such that $X+Y=1$ in any phase. There are three possible phases in this system: crystals of α , crystals of β , or liquid (L). Melt of eutectic composition is represented by L_e for which $X=X_e$. There are five possible phase assemblages in this system: L, $\alpha+L$, $\beta+L$, $L_e+\alpha+\beta$ or the crystalline assemblage $\alpha+\beta$. The phase diagram and thermochemistry are defined by specification of X_e , T_e , the fusion enthalpies of α (Δh^α) and β (Δh^β) at their respective melting temperatures of $T_{m.p.}^\alpha$ and $T_{m.p.}^\beta$ and a single constant isobaric specific heat for crystals of either α or β (C_S) and for melt (C_L). The liquidii of the toy model are linearized in T-X space. This approximation makes little difference to any of the basic insights gained by study of the toy model regarding magma hybridization. The characteristic concave-down shape of liquidii could easily be captured using fusion entropies and taking account of the entropy, volume and enthalpy of mixing (i.e., non ideality) as in a standard liquidus curve calculation. However, the algebra becomes more cumbersome and nothing new is gained conceptually. Hence the two branches of the liquidii in T-X space are linearized such that for $X < X_e$,

$$T_{\text{liquidus}} = \left(\frac{T_e - T_{m.p.}^\alpha}{X_e} \right) X + T_{m.p.}^\alpha \quad (\text{A1})$$

whereas for $X > X_e$,

$$T_{\text{liquidus}} = \left(\frac{T_e - T_{m.p.}^\beta}{Y_e} \right) Y + T_{m.p.}^\beta \quad (\text{A2})$$

Characterization of the initial state of **M** and **R** magmas

Once the phase diagram, magma thermochemical properties and Φ are defined, five additional parameters are required to initialize the magma hybridization. The initial state of **M** and **R** are defined by specifying their bulk compositions (X_o^M, X_o^R), temperatures (T_o^M, T_o^R) and the fraction of **M** magma in the magma mixture (f_o). Given bulk compositions and initial temperatures of **M** and **R**, phase assemblages in each can be determined from the phase diagram (lever rule) and liquidii T-X relations. Once the phase assemblage and liquid compositions (if applicable) for **M** and **R** are known, the specific enthalpy of each can be calculated and, by appropriate weighting, the specific enthalpy of the mixture computed. When magma hybridization is isenthalpic (R-hybridization), the final enthalpy of the hybrid magma is identical to the sum of the mass weighted specific enthalpies of **M** and **R**. When the process is diabatic (RFC-hybridization), then the specific enthalpy of **H** magma is Φ times the initial enthalpy of **M+R**, the remainder $(1-\Phi)$ being dissipated externally. The starting assemblage of **M** and **R** depend on their bulk composition and initial temperature and hence expressions for the specific enthalpy take into account phase state and proportions. The relevant expressions are collected in Table A1, which give the contributions that **M** and **R** make to the specific enthalpy of the mixture. As one example of many initial possibilities, consider initial **M** magma of bulk composition $X_o^M < X_e$ is just at its liquidus (all melt) and that **R** magma of composition $X_o^R > X_e$ lies at a temperature between the β -saturated liquidus and the eutectic. In this case, **R** is a two-phase assemblage of $\beta + L$ whereas **M** is a crystal-free liquid denoted by the subscript $L\alpha$ in Table A1. In this case, the initial specific enthalpy of the mixture is given by $h_o = h_{L\alpha}^M + h_{\beta+L}^R$ which from Table A1 is:

$$h_o = f_o \left[C_S T_o^M + \Delta h^\alpha + X_o^M (\Delta h^\beta - \Delta h^\alpha) + \Delta C \left(X_o^M (T_{m.p.}^\alpha - T_{m.p.}^\beta) + (T_o^M - T_{m.p.}^\alpha) \right) \right] \\ + (1 - f_o) \left[C_S T_o^R + \left(\frac{Y_o^R}{Y_o^{R\ell}} \right) \Delta h^\beta + Y_o^R (\Delta h^\alpha - \Delta h^\beta) + \Delta C \left(Y_o^R (T_{m.p.}^\beta - T_{m.p.}^\alpha) + \left(\frac{Y_o^R}{Y_o^{R\ell}} \right) (T_o^R - T_{m.p.}^\beta) \right) \right] \quad (A3)$$

Any combination of states of **M** and **R** can be constructed using appropriate pairs from Table A1. The composition of the melt along the liquidus in eq (A3) is found from eq (A2) by setting T_{liquidus} equal to T_o^R and solving for $Y_o^{R\ell}$, the composition of melt along the β -saturated liquidus. As a second example, consider a β -saturated **M** magma that receives stopped wholly crystalline blocks of **R** of assemblage ($\alpha+\beta$) In this case, the initial specific enthalpy of the mixed magma is:

$$f_o \left[C_S T_o^M + \left(\frac{Y_o^M}{Y_o^{M\ell}} \right) \Delta h^\beta + Y_o^M (\Delta h^\alpha - \Delta h^\beta) + \Delta C \left(Y_o^M (T_{m.p.}^\beta - T_{m.p.}^\alpha) + \left(\frac{Y_o^M}{Y_o^{M\ell}} \right) (T_o^M - T_{m.p.}^\beta) \right) \right] \\ + (1 - f_o) [C_S T_o^R] \quad (A5)$$

which accounts for the β -phase saturation of **M** and the crystalline nature of **R**. To complete initialization of the system, the bulk composition of the hybrid magma is simply found as the mass-weighted average of **M** and **R** bulk compositions according to:

$$X^H = f_o X_o^M + (1 - f_o) X_o^R \quad (A4)$$

This completes characterization of the initial state when magmas **M** and **R** are mixed and hybridized (i.e., reach thermodynamic equilibrium).

Characterization of the final phase assemblage of hybrid (**H**) magma

The specific (per unit mass) enthalpy h of the **H** magma is given by

$$h^H = \Phi (h^M + h^R) \quad (A5)$$

The weighted contribution of **M** and **R** to the mixture are given in Table A2. The parameter Φ defines the type of hybridization. If $\Phi = \text{one}$, the mixing is isenthalpic

(adiabatic) also called R-hybridization. If $0 < \Phi < 1$, the mixing is diabatic and termed RFC-hybridization. There are five possible state assemblage outcomes when **M** and **R** hybridize. The final hybrid magma can consist of either Liquid (L), α crystals + liquid ($\alpha+L$), β crystals + liquid ($\beta+L$), eutectic liquid+ α crystals+ β crystals ($L_e+\alpha+\beta$), or crystals of α and β ($\alpha+\beta$). Solid phase identities, liquid composition and temperature are found by comparing the specific enthalpy of **H** magma computed from Eq. (A5) to enthalpy limits defined *a priori* for the five possible outcomes. These phase assemblage limits in h-T space are depicted schematically in Figure 3 of the text. Once X^H is given, the h-T diagram for that composition can be determined using the expressions given in Table A2. The five possible final state assemblages occupy distinct regions on the h-T diagram. There are three special enthalpies on this diagram denoted h_{max} , h_{mid} and h_{min} . These values separate phase assemblages. For example, when the specific enthalpy of hybrid magma h^H of bulk composition X^H exceeds h_{max} , then the final hybridized magma must lie in the L field on the phase diagram. Similarly, if $X^H > X_e$ and $h_{mid} < h^H < h_{max}$, then hybrid magma will consist of $\beta+L$ or if $X^H < X_e$, and $h_{mid} < h^H < h_{max}$, the H magma assemblage is $\alpha+L$. When the hybrid magma enthalpy lies in the range $h_{min} < h^H < h_{mid}$, then the assemblage is $L_e+\alpha+\beta$ and the amount of eutectic liquid is determined by enthalpy balance. In this case, the temperature is identically equal to T_e , the eutectic temperature. Finally, if $h^H < h_{min}$, the assemblage is a mixture of α and β crystals in proportions dictated by the lever rule and the temperature is less than T_e . In summary, in order to find the final state of the hybrid magma, the value of h^H is compared to the ranges given in Table A2 to discover which of the five possible assemblage outcomes is relevant.

Characterization of the temperature and phase composition(s) of hybrid (H) magma

Once the phase state or outcome is known by comparing h^H to the limits specified in Table A2 (see Figure 3), the final state of hybrid magma can be determined. The state depends first on comparison of X^H with X_e and then on the value of h^H . The conditions and final state values are given in Table A3 when $X^H < X_e$,

Table A4 is valid when $X^H > X_e$ and Table A5 is valid when $X^H = X_e$ (exactly). Note that in the latter case, the $\alpha+L$ or $\beta+L$ fields are not possible.

As a summary example, consider the possibilities when $X^H < X_e$. From the phase diagram, the state of **H** magma can be one of four states (L , $L+\alpha$, $L_e+\alpha+\beta$, $\alpha+\beta$). If $h^H > h_{\max}$, then **H** is a single phase melt of composition equal to the bulk composition and its temperature is given from the expression in the first row of Table A3. If instead, $h_{\text{mid}} < h^H < h_{\max}$ then the **H** magma consists of liquid plus α crystals. Simultaneous solution of the two expressions in row three of Table A3 gives T^H and the composition of melt in **H** magma ($X^H = X^{H\ell}$) in the $L+\alpha$ field, thereby defining the appropriate tie line. If $h_{\min} < h^H < h_{\text{mid}}$, the state is defined by the invariant point assemblage of $L_e+\alpha+\beta$. In this case, $T^H = T_e$ and $X^{H\ell} = X_e$. The mass fractions of L_e , α and β crystals are given in row 4 of Table A3. Finally, when $h^H < h_{\min}$, the assemblage is wholly crystalline ($\alpha+\beta$ crystals) in proportions given in the fifth row of Table A3. Table A4 gives analogous solutions when $X^H > X_e$ and Table A5 is appropriate when $X^H = X_e$, exactly.

Table A6 collects thermodynamic parameters that approximately model the system $\text{CaMgSi}_2\text{O}_6$ - $\text{CaAl}_2\text{Si}_2\text{O}_8$ at 10^5 Pa (1-bar). The toy code can be found at the following URL <http://magma.geol.ucsb.edu/>. Once downloaded, a user is free to change any of the thermodynamic parameters and run computations for any binary eutectic system with known parameters.