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Chapter 7

ASPECTS OF MAGMA TRANSPORT

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This paper is dedicated to the memory of Christopher Goetze.
His death is a sorrowful loss to all of those who knew him and some who did not.
His contributions to the study of rock deformation will long serve as a
benchmark for excellence. Progress in the rheology of the mantle
has been set back by his loss.

INTRODUCTION

The problem of the transport or mobilization of magma is central not only to igneous petrogenesis but also to the evolution and dynamics of the Earth. The generation and emplacement, for instance, of roughly 15 km^3 per year of basaltic magma along the axes of diverging plate margins is a first-order, petro-tectonic process. A corollary of the implied mass-transfer process is the transport of large quantities of heat from the interior of the Earth. The heating (from 0°C to 1,200°C) and complete fusion of 15 km^3 of magma requires an amount of energy roughly equivalent to about 10% of the annual average heat flow out of the Earth. If mean mantle temperatures were higher in the past, as some suggest (McKenzie and Weiss, 1975), then one might suspect that magma production rates also assumed greater values. No matter how volumetric eruption rates may have varied in the past, the conclusion that magmatism has played a significant role in the geological evolution of the Earth is inescapable. The almost total restriction of high-temperature periodotitic komatiitic lavas to the Archean ($>2.5 \times 10^9$ yrs B.P.) illustrates the importance of this view in

terms of the thermal history of the Earth.[1] Before one may use volcanic and plutonic rocks to constrain the composition and thermal evolution of the crust and mantle, however, the "petrological inverse problem" must be solved. That is, it must be shown how the final volcanic or plutonic product is influenced by the path traversed by the melt, as well as the composition and environmental conditions prevailing at the source.

The term path is used here in an extended thermodynamic sense. It refers not only to the temporal variation of intensive variables such as T, P_{total}, a_{SiO_2} and f_{O_2} but also to the gradients (spatial variation) of these variables. The temperature gradient within a mantle diapir rising through relatively cool lithosphere will, for example, be related to the rate of heat loss to the surroundings. Similarly, pressure and chemical potential gradients will influence magma flow and mass-transfer rates. It is the purpose of this paper to outline some aspects of the "petrological inverse problem" by analyzing mobilization mechanisms in terms of the transport phenomena governing the evolutionary path magmatic systems follow.

The application of transport phenomena to petrological systems consists of three main parts: momentum transport (e.g., viscous flow in conduits and diapirs, segregation and flow of melt in crystal mushes), energy transport (heat conduction, buoyancy convection, viscous dissipation) and mass transfer (diffusion and advection-aided diffusion, infiltration, nucleation and growth of crystalline phases or vapor bubbles). Natural mobilization mechanisms involve a simultaneous superposition of all three types of transport. For example, the estimation of the melting rate of a crystal is not possible unless some auxiliary assumptions about the dynamic and thermal state of the system are specified. Contrast the growth rate of a crystal in an immobile versus mobile body of magma. A primary factor governing interface-controlled crystal growth rates is the degree of supercooling (Kirkpatrick, 1974). The degree of supercooling, ΔT, may be taken as the difference between the equilibrium liquidus temperature of the crystallizing phase and the temperature at the crystal-melt interface. When a relatively large differential velocity exists between melt and crystallite, the fluid is capable of continuously sweeping away the heat produced by the crystallization process, so that the interface temperature and hence degree of supercooling is dependent on the cooling of the igneous body as a whole and not any local interface effects. At the other extreme is the case in which the buildup of crystallization heat around the growing crystal increases the local interface temperature, and therefore decreases the extent or degree of supercooling, other factors remaining the same. For ΔT's less than about 50 to 100°C, growth rates increase with an

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[1] See Upadhyay (1978) for an account of peridotitic komatiite flows in an Ordovician ophiolite suite from Newfoundland.

increasing degree of supercooling (Kirkpatrick 1974), and so, within this temperature range, a quickly settling crystal grows faster than one moving more slowly. In semiquantitative terms, note that the ratio of heat or mass transport by advection (bulk flow) to that by heat conduction or mass diffusion is given roughly by u l/κ or u l/D respectively. In these expressions, u and l represent a characteristic velocity and length, κ the thermal diffusivity, and D the mass-diffusion coefficient. With typical values of $u = 10^{-2}$ cm s^{-1}, $\kappa = 10^{-2}$ cm^2 s^{-1}, $l = 10^2$ m and $D = 10^{-7}$ cm^2 s^{-1}, the growth (or dissolution) rate of a crystal in a magma can vary considerably depending on kinematic and thermal boundary conditions. Because of the large number of degrees of freedom associated with irreversible processes, such as mass, heat, and momentum transfer, the elucidation of mobilization mechanics promises to be a complicated business.

In the context of this paper, mobilization refers to the collection of processes that influence the final state of a magmatic system. These processes include melt generation, segregation, ascent, and emplacement or eruption. An important aspect of this paper is the realization that the genetic and transport processes associated with magmatism are best viewed as operating on a variety of concurrent temporal and spatial scales. Virtually all mobilization or transport mechanisms are evidence of mechanical or thermal instabilities. From a dynamic point of view, the eruption or emplacement of a quantity of magma is but the last in a long chain of evolutionary instabilities that characterize most chemical and mechanical systems displaced far from equilibrium (Nicolis et al., 1975; Prigogine and Lefever, 1975).

New and sometimes surprising patterns of spatial or temporal organization are exhibited by many nonequilibrium systems. An example from biology is the process of cell division. It has been suggested (Nicolis and Prigogine, 1977) that division occurs when the concentration of some metabolite reaches a threshold concentration that depends in a complicated fashion on the interplay of the nonlinear kinetics of metabolite production, the processes of mass diffusion, and the size of the cell. Cell division in this view is seen essentially as an instability or bifurcation marking a change from one organizational biochemical regime to another. An example perhaps more germane to magma transport is the onset of convection in a fluid confined between two horizontal rigid walls at different temperatures. If the upper plate is maintained at a fixed temperature and the temperature of the bottom plate is increased, a critical value of temperature difference is eventually reached, and the fluid spontaneously overturns until the temperature gradient is erased. The magnitude of the critical temperature difference depends on a number of parameters, including the thermal diffusivity, viscosity, and isothermal expansivity of the melt, the distance between the plates, and the magnitude and direction

of the acceleration due to gravity vector. Below the critical value of the temperature gradient, the fluid experiences no movement and heat is simply transferred by conduction. Above this value, a new pattern of spatial organization emerges (i.e., a convection cell) that is radically different from the previous stationary state.

Another possible example of organization in non-equilibrium systems is the development of oscillatory zoning in feldspars. Here, perhaps, the balance between reaction at the interface and long-range diffusion (of mass, heat, or both) is responsible for the pattern of a spatially varying mole fraction of albite component. The interplay of competing kinetic or transport effects is very much the theme in the application of transport phenomena to petrogenesis.

The purpose of this paper is to survey some of the wide spectrum of transport processes and to give a brief and somewhat simplified (perhaps naive) account of the mechanics involved. This survey makes no pretense of being exhaustive; in recent years great effort has been applied towards an understanding of the transport and kinetic phenomena that constrain petrogenetic theories (see Hofmann et al., 1974; Yoder, 1976; and this volume for appropriate examples). The topics discussed here represent those familiar to the author that seem generally relevant to a wide range of petrologic phenomena. The aim is simply to point out the diversity of transport mechanisms and show that, in fact, these mechanisms reveal some common characteristics when they are analyzed from a dynamic point of view.

MOBILIZATION MECHANICS

In the following sections an attempt is made to give a brief introduction to the mechanics of magma mobilization and flow. The order of topics corresponds in a general way with the *inversion* of the route magma travels from source to the surface or high-level emplacement zone. Although there may be no real advantage associated with the order of topics chosen here, this author finds it easier to physically visualize processes associated with magma transport in the upper crust. This is in part due to the relative accessbility of the crust for observation as compared to, for instance, the mantle 60 or 70 km beneath Hawaii. The discussion commences, consequently, with a brief analysis of the mechanics of fluidization phenomena. This analysis finds application to situations as varied as the eruption of a basalt exsolving a volatile phase and the emplacement of diatremes within the crust. A discussion of the role played by propagating cracks in the upward movement of magma through regions undergoing

penetrative deformation follows the section on fluidization. Viscous flow in conduits where pressure gradients or thermal and compositional buoyancy effects drive the flow are then discussed in relation to the conditions for turbulence. The paper ends with an account of the processes that occur at depth, an introduction to the thermodynamics of adiabatic and nonadiabatic magma ascent, and consideration of the factors that influence melt segregation at depth.

A summary section focuses on the critical role played by the rheology of both mobilized magma and the environment it passes through. Rhelogical behavior is dependent on pressure, temperature, and composition; the gradients of these; and the crystallinity and rate of deformation of both the magma and the surrounding mostly crystalline materials.

FLUIDIZATION PROCESSES

Although magma is generally thought to consist primarily of a silicate liquid phase, its eruption as a lava can be greatly facilitated by the development of a volatile phase. Estimates of the pristine H_2O and CO_2 contents of basaltic magmas (Moore, 1970; Moore et al., 1977) suggest that most if not all melts attain volatile saturation during the last few kilometers of ascent. The exsolution of a gas phase in response to a decrease in P_{total}, and an increase or decrease in T, depending on the sign of the enthalpy of the exsolution reaction, could generate volatile pressures sufficient to fluidize a system. From what is known about the chemistry and phase relations in silicic ignimbrites and ash flows, H_2O would seem to be the dominant volatile species (Ross et al., 1961, Smith, 1960; Tazieff, 1970; Pai et al., 1972). The emplacement of both kimberlites (Dawson, 1967; Hearn, 1968; Harris et al., 1970; McGetchin et al., 1973a, 1973b) and the brecciated minettes (lamprophyres) of some volcanic necks (Currie et al., 1970; Turner and Verhoogen, 1960, p. 250–256) have also been attributed to fluidization processes. In the case of the alkaline and carbonatitic rocks, a CO_2-rich gas phase is thought to have been the propellant; in some cases 10% to 20% by mass of volatiles may be retained in carbonate and hydrous phases (Harris et al., 1970).

The magnitude of the ascent velocity of the low-viscosity phase (liquid or supercritical fluid) in a fluidized system may be estimated by consideration of the physics of flow through packed beds.

Imagine a two-phase (liquid-solid or gas-solid) system, in which the less viscous phase is flowing through an immobile packed bed. In the case of a kimberlitic diatreme, for example, the packed bed would be represented by a poorly sorted mixture of xenoliths, megacrysts and brecciated country rock. It has been shown (Kay, 1968; Soo, 1967) that the pressure drop for

flow through a packed bed of uniform solid particles is related to the mean fluid flow velocity, u_m, by

$$\Delta p = 3k_f\lambda \frac{1-\varepsilon}{\varepsilon^3}\frac{L}{D}\rho u_m^2, \qquad (1)$$

where k_f, λ, ε, L and D_s represent the friction coefficient, particle shape factor,[2] bed porosity, length of the fluidized zone, and nominal diameter of uniform equivalent-volume spheres. The magnitude of k_f depends on whether the flow is in laminar or turbulent motion. In the laminar region, the Blake-Kozeny equation is found empirically to be applicable. This relation gives (for uniform spherical particles)

$$u_m = \frac{\Delta p}{L}\frac{D_s^2}{150\eta}\frac{\varepsilon^3}{(1-\varepsilon)^2}. \qquad (2)$$

Equation (2) is thought to be valid for laminar flow where the relation

$$\frac{\rho D_s u_m}{\eta(1-\varepsilon)} < 10 \qquad (3)$$

holds (Bird et al., 1960). In turbulent flow through packed beds, the Burke-Plummer relation is valid when

$$\frac{\rho D_s u_m}{\eta(1-\varepsilon)} > 1{,}000. \qquad (4)$$

The velocity in turbulent flow may therefore be written

$$u_m^2 = \frac{4}{7}\left(\frac{\Delta p}{L}\right)\frac{D_s}{\rho}\frac{\varepsilon^3}{(1-\varepsilon)}. \qquad (5)$$

Of course, the solid particles in a diatreme or in the throat of a volcanic pipe, or the gas bubbles in a basalt, are not of uniform size. For a bed of mixed particles (either crystals, melt droplets, or gas bubbles) the pressure drop may be represented by

$$\Delta p = 3k_f\lambda \frac{1-\varepsilon}{\varepsilon^3}\frac{L}{D_m}\exp(\sigma^2)u_m^2, \qquad (6)$$

assuming a log-probability distribution of particles, where D_m is the median particle diameter as determined, for example, by sieving measurements.

As u_m increases, there is a value of the flow rate, denoted by u_{crit}, such that the bed of solid particles will be supported by the high flow rate of the

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[2] For spherical particles, $\lambda = 1$; all other shapes have $\lambda > 1$.

fluid stream. When $u_m = u_{crit}$, the bed or tube is said to be fluidized; individual xenoliths or clasts have freedom of movement in the gas stream. With uniform fluidization, the flow is homogeneous and independent of time (i.e., the flow is steady); however, heterogeneities associated with non-uniform xenolith size and shapes probably induce channeling effects, by which the flow becomes unsteady. As the velocity of flow is increased beyond u_{crit}, entrainment of xenoliths will occur and they will be swept upwards. In some instances, geological evidence indicates that critical velocities have been exceeded, because known stratigraphic relations imply a net upwards motion for some xenoliths (McGetchin et al., 1973a, 1973b, 1975).

One may compute an approximate value of u_{crit} by noting that, at the point of fluidization, the pressure given by (1), (2), (5) or (6) must be balanced by the weight of the solid particles supported in the gas stream. This latter quantity is

$$\Delta p = (1-\varepsilon)(\rho_s - \rho_f)gL. \qquad (7)$$

If one assumes uniform-size particles, the critical mean velocity (u_{crit}) is

$$u_{crit} = \left\{\frac{\Delta\rho\, gD_s(\varepsilon^*)^3}{3\rho_f k_f\lambda}\right\}^{1/2}. \qquad (8)$$

In the laminar regime, (8) reduces to

$$u_{crit} = \frac{\Delta\rho\, gD_s^2(\varepsilon^*)^3}{150\eta(1-\varepsilon^*)}, \qquad (9)$$

whereas for turbulent flow

$$u_{crit} = \frac{2}{7}\{D_s(\varepsilon^*)^3\,\Delta\rho\, g\}^{1/2}. \qquad (10)$$

In (8), (9) and (10), ε^* signifies the porosity of the bed at the critical velocity and $\Delta\rho$ is the density difference (taken as positive) between the solid (ρ_s) and fluid (ρ_f) phase. Although clearly a simplification, the order of u_{crit} may be calculated by the appropriate use of (9) or (10). It should be carefully noted that, while these relations give a rough idea of how u_{crit} depends on the other parameters, the precise numerical value of u_{crit} may in some cases depend on other factors not discussed here—for example, wall effects, unsteady flow, nonspherical and graded particles, fall of particles in non-newtonian melts and grain-dispersive pressures. A more complete summary of fluidization phenomena is currently in preparation by this author.

As an example of the use of these relations consider two illustrative cases: (a) gas-fluidized bed (e.g., CO_2 in a kimberlite, H_2O in an ash flow or a CO_2–H_2O mixture in a vesiculating basalt); and (b) a liquid-fluidized

bed (e.g., a xenolith-packed basalt flow). For (a), the following parameters are assumed.

$$\Delta\rho = 3 \text{ gm cm}^{-3} \qquad D_s = 50 \text{ cm}$$
$$\eta = 10^{-2} \text{ gm cm}^{-1} \text{ s}^{-1} \qquad \varepsilon^* = 0.5$$

From Equation (10) one may compute $U_{crit} = 1 \text{ m s}^{-1}$ For (b) we assume

$$\Delta\rho = .5 \text{ gm cm}^{-3} \qquad D_s = 25 \text{ cm}$$
$$\eta = 10 \text{ gm cm}^{-1} \text{ s}^{-1} \qquad 0.2,$$

and compute from Equation (9)

$$u_{crit} = 0.2 \text{ cm s}^{-1}$$

The physical parameters used to make these estimates have been taken from a number of sources (McGetchin et al., 1973a, 1975; Bird et al., 1960; Carmichael et al., 1977). For an alternative view of explosive volcanism, the reader is referred to the thought-provoking works of Bennett (1971, 1972).

In the case of gas exsolution from a melt, these simple equations are applicable if modified to account for the details of the kinetic problem concerning nucleation and growth rate of gas bubbles from the melt. The interested reader is referred to Sparks (1977) for a more complete discussion. Vapor bubbles in a viscous medium tend to increase dynamic viscosities, and there is a concurrent transfer of heat between solid and fluid. Some of these complications have been discussed with specific reference to ash-flow eruptions by Pai et al. (1972).

In terms of the applicability of fluidization phenomena to igneous petrogenesis, it should be noted that virtually all magmas contain some volatile constituents capable of exsolution at low pressure. For the more voluminous magma types (e.g., mid-ocean ridge basalts, alkali basalts of the oceanic islands, calc-alkaline island-arc volcanics and related plutonic rocks, and continental silicic volcanic rocks), however, the generation of a fluid phase by exsolution occurs at relatively shallow (crustal) depths. A basaltic melt with 1% wt. H_2O reaches water saturation at a pressure equivalent to a depth of roughly one kilometer.

More silicic melts are thought to be slightly more hydrous. Maaloe and Wyllie (1975) believe it uncommon for the water content of large granitic magma bodies to exceed about 2%. Eggler (1972) experimentally showed that the presence of orthpyroxene phenocrysts (as opposed to olivine) implies a maximum H_2O content of 2.5% in a Paricutin andesite. Precise values of the saturation depths at which fluidization is at least potentially applicable may be determined using solubility data from many sources,

notably including Burnham and Davis (1971) for H_2O, and Eggler (1978) and Mysen (1977) for CO_2 and CO_2-H_2O mixtures. For another view of water and other volatile concentrations in melts, the reader should consult the works of Anderson (1974a, 1974b, 1975).

The dynamics of gas-crystal fluidization is also relevant to the eruption of kimberlitic and other diatremic rocks. Although these rocks are relatively rare, an understanding of their mechanical and thermal histories would provide insight about temperatures and the distribution of volatiles within the Earth. Complicated three-phase fluidization regimes may be applicable to the eruption of some silicic crystal-tuffs (e.g., Lipman, 1967). Studies regarding the dynamics of three-phase fluidization are not as numerous as two-phase ones, but are becoming more frequently the subject of experimental and theoretical investigations.

In conclusion, the need for careful field descriptions of the abundance, size, and distribution of the various inclusion types in rocks emplaced or presumed to have been emplaced by fluidization processes is emphasized. These data, when combined with estimates of volatile fugacities as calculated from thermodynamic expressions or phase equilibria studies (using compositions of associated phenocryst phases), are the basic information needed to estimate minimum fluidized eruption rates from Equation (6) or its more sophisticated and derivable analogues.

MAGMA TRANSPORT BY CRACK PROPAGATION

The emplacement or eruption of magma implies bulk movement of melt through the Earth's upper mantle and crust. The course rising magma follows is influenced by the magnitude and variation of regional tectonic stresses (e.g., see Jackson and Shaw, 1975; Jackson et al., 1975; Fiske et al., 1972). The presence of a low-viscosity fluid phase (e.g., a volatile phase or magmatic fluid) at a total pressure roughly equal to P_{lith} will greatly reduce the overall strength of the surroundings.[3] Migrating melts may also be thermally and chemically active. A volatile-enriched low-viscosity melt may act as a corrosive agent on the surroundings, and, with the aid of pressure forces, be injected along pre-existing or newly-formed zones of weakness. One limit to magma ascent rates is the velocity of propagation of a weakened fracture zone. The rate of crack propagation may be computed with some confidence in certain simplified cases (e.g., a perfectly elastic homogeneous substance in a homogeneous stress field); however,

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[3] The term lithostatic pressure refers to the pressure at the bottom of a column where the density of the overburden has the average value $\bar{\rho}$. Then $P_{lith} = \bar{\rho}gz$ where z is the thickness of the overburden.

the rates of propagation of magma-filled cracks can only be roughly estimated given the present state of the art.[4]

Aki et al. (1977) proposed a model for the mechanism of magma transport in the upper crust based on migrating-fluid (magma) tensile cracks driven by an excess of magma pressure over lithostatic pressure. They claim that this mechanism can generate seismic waves by a succession of jerky crack extensions when fracture strengths of surrounding country rocks vary spatially. Their model leads to predictions regarding the spectrum of seismic-energy radiation from propagating cracks approximately in accord with the spectral features observed for volcanic tremor. It would seem then that, at least at shallow depths, excess magmatic pressure in magma-filled cracks is an important driving force for transport processes. The ultimate origin of the implied excess magma pressures is not entirely clear· if exsolution of volatiles is responsible for the pressure differential, there should be a correlation between the temperature and volatile content of a magma with the depth at which transport by the mechanism of Aki et al. (1977) becomes operative. As will be pointed out in a later section, even when no excess of magmatic over lithostatic pressure exists, fractures may open and be injected with melt, provided certain relations between the principal effective stresses hold. The important role of local and regional stress states to melt transport warrants a discussion of the factors that govern the formation, configuration and propagation rates of magma-filled cracks.

Griffith Theory of Brittle Failure

The motivation for studying and applying the Griffith theory of brittle rock failure is firmly anchored in the concept of the effective stress and the role of fluid pressure in formulating quantitative failure theories. As previously mentioned, the presence of a fluid phase (e.g., volatile or melt phase) greatly reduces the overall strength of a medium, thereby increasing the probability of the nucleation and propagation of a crack or zone of weakness within that material. An elaboration of this idea is presented in Appendix I. The sections that follow will assume basic familiarity with the ideas presented in that Appendix.

Much has been written concerning quantitative criteria for brittle failure in rocks. Perhaps one of the most successful models is that attributed to Griffith (1921, 1925), as modified and discussed by many other distinguished geologists (for example; Ode, 1957; Brace, 1960; Murrell,

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[4] Detailed seismicity studies of earthquake swarms within the mantle in regions where other geophysical and geological evidence suggests the presence of mobilized melt may make it possible to estimate flow rates if accurate locations can be determined and if it is assumed that the propagation of melt filled cracks is accompanied by seismic signals (see Aki et al., 1977; Klein et al., 1977).

1964, 1965; McClintock and Walsh, 1962; Secor, 1965). The Griffith theory is essentially based on the supposition that all solids are filled with tiny cracks. When differences in the principal stress deviator components (see Appendix I) exceed a certain critical value, high tensile stress will occur near the ends of the Griffith cracks even when all the principal stress deviators are compressive (a compressive normal stress is taken as positive). Failure occurs by the spontaneous growth of a Griffith crack in response to sufficiently large tensile stresses developed at the tip of a suitably oriented flaw.

A particularly convenient way to represent the stress conditions for failure is by means of a stress envelope for rupture on a Mohr diagram (see Shaw, this volume). That is, on a plot of τ versus σ_{eff}, where σ_{eff} is the effective stress (Appendix I), there is a locus of (τ, σ) points, represented by some equation of the form $\tau = f(\sigma)$, that may be taken as the criterion for failure. The envelope predicted by the Griffith theory is of the form (for plane stress)

$$\tau^2 - 4K\sigma - 4K^2 = 0, \tag{11}$$

where $-K$ is the tensile strength and τ and σ represent the shear and normal components of stress on the plane of the most critically oriented cracks (Secor, 1965; Tremlett, 1978).

Natural fracturing can occur in response to either tensile or compressive principal stresses (i.e., anywhere along the failure envelope). If the magnitude of the isotropic fluid pressure or magma pressure (p_f) is equal to the ambient lithostatic pressure, fractures will open up only if the normal stress across the fracture surface is tensile. Stated another way, when $p_f = p_{lith}$, injection of magma can occur only for values of the principal stresses corresponding to the tensile (negative) part of the Mohr envelope. These stress states correspond to the formation of either tension fractures perpendicular to the minimum principal stress axes, or to conjugate shear fractures inclined to the directions of principal stresses such that the acute angle of conjugate intersection is bisected by the direction of $\sigma_{I, eff}$. It is important to note that, if $p_f = p_{lith}$, then shear fractures will open only if the stress circle at failure intersects the Mohr envelope at a point corresponding to a tensile (negative) normal stress. If, on the other hand, p_f is greater than the average of the principal stresses, shear fractures across which a *compressive* normal stress acts may be injected with magma.

The failure criteria for tension fracturing are that

$$\sigma_{III, eff} = -K, \tag{12a}$$

and that

$$\sigma_{I, eff} \leq -3K. \tag{12b}$$

Note that the effective stress is defined according to

$$\sigma_{III, eff} = \sigma_{III} - p_f.$$

In regions of the Earth where $\sigma_{I, eff} > 3K$, tension fractures could not open. If σ_I is taken as coincident with the vertical direction, the form of an igneous intrusion would be a vertical magma-filled crack (e.g., a dike). In the case when σ_I is horizontal, a sheet-like body would form. The interested reader is referred to Roberts (1970) for an account of the various configurations of magma intrusions into brittle rocks, and their dependence on the stress field.

In shear fracture, Murrell (1964) cites the criteria for open fractures to be

$$-K < \sigma_{III, eff} < 2(1 - \sqrt{2})K \tag{13}$$

$$3K < \sigma_{I, eff} < 2(\sqrt{2} + 1)K, \tag{14}$$

where $-K$ represents the tensile strength.

Values of K are rather small, varying between several tens to perhaps several hundred bars (Brace, 1960 and 1964; J. Suppe, personal communication) and are sensitive to the length of the Griffith crack (Haward, 1970). The variation of K with temperature is not well-documented, but intuitively it seems that K would decrease at elevated temperatures. The experimental demonstration of decreasing K with increasing temperature is, no doubt, complicated by the increased tendency for ductile behavior at elevated temperatures.

The conditions for failure with melt propagation require effective principal deviatoric stresses on the order of several hundred bars. This constraint does not represent an effective inhibition to failure and melt intrusion in the Earth. independent evidence, including the magnitude of geoidal undulations and stress drops accompanying earthquakes, indicates that nonlithostatic deviatoric stresses of several hundred bars may develop within the Earth. Direct measurements of $\sigma_{I, eff} - \sigma_{III, eff}$ within the upper parts of the crust support this view (Haimson, 1977).

Rates of Crack Propagation

The fracture process easiest to visualize is the catastrophic failure of a brittle substance (e.g., breaking a beer bottle) in which the velocity of crack propagation rapidly approaches $\frac{4}{10}$ of the compressional wave velocity of the material. Many materials, however, undergo an earlier phase of slower ($\sim 10^{-3}$ m s^{-1}) cracking before final catastrophic failure. The rate of subcritical or non-catastrophic crack growth is influenced by factors such as

the temperature and chemistry of the crack environment (Anderson and Grew, 1977).

Fast-Cracking

Griffith (1921) was the first to apply the concept of conservation of energy to determine the conditions for spontaneous growth of a crack in terms of the tensile stresses acting at its tip. His result for the threshold tensile stress, found by minimizing the total energy of the system with respect to the length of the crack, is given by

$$\sigma_{crit} = \left\{ \frac{4\rho V_s^2 \gamma (1 + v)}{\pi a (1 - v)^2} \right\}^{1/2}, \tag{15}$$

where ρ, V_s, γ and v represent the density, shear wave velocity, surface energy and Poisson's ratio for the material, and a and σ represent the half-length of the crack and the tensile stress at the tip of the crack, respectively. For a typical *crystalline* silicate (a dunite from Twin Sisters, Mt. Washington) representative values are as follows (Birch, 1966; Swalin, 1970; Verhoogen et al., 1970):

$$\rho = 3.31 \text{ gm cm}^{-3} \qquad V_s = 4.9 \text{ km s}^{-1}$$
$$\gamma = 10^3 \text{ erg cm}^{-2} \qquad v = .24.$$

From Equation (15), the critical stress associated with a 1 cm long crack is about 50 bars. It might be intuitively expected that a melt-filled crack requires smaller stresses for propagation. Aki et al. (1978) analyzed seismograms related to the propagation of seismic waves through the partially frozen lava lake at Kilauea Iki. Their data indicate severe attenuation of shear waves traveling through the buried magma body. Their value of $V_s = .2$ km s^{-1} may be combined with the values

$$\rho = 2.7 \text{ gm cm}^{-3} \qquad \gamma = 10^3 \text{ erg cm}^{-2} \qquad v = .5$$

to give a critical stress of 3 bars necessary for the propagation of a 1 cm long crack through a partly consolidated magma. Although these figures should be considered as gross estimates, the calculations suggest that crack propagation may be initiated in crystal mushes at stresses at least an order of magnitude smaller than in corresponding crystalline materials.

Extensions of the Griffith theory that take account of kinetic energy terms have been proposed (Lawn and Wilshaw, 1975; Roberts et al., 1954; Anderson, 1959). The calculation of fast-crack propagation rates is possible using expressions developed by these authors. A noteworthy result is that, after the critical stress has been attained, the crack accelerates and very rapidly attains a propagation rate equal to about $\frac{2}{5}$ the P-wave velocity of the material.

The slow-cracking process, which is probably more relevant to the magma transport problem, is more realistic and consequently more difficult to analyze. As summarized by Anderson and Grew (1977),

.. many materials exhibit time-dependent fracture, that is, when they are subjected to a static load, they fail after some time interval that depends inversely on the magnitude of the load. Griffith's (fast-cracking) theory fails to account for this phenomenon, because it predicts that a stress only infinitesimally less than the critical value would be sustained indefinitely.

The effects of temperature and the presence of chemically reactive fluids are particularly important in determining the rate of slow-crack growth. In stress corrosion cracking,[5] the limiting crack velocity is controlled by diffusion and viscous processes in the fluid at the tip of the crack.

Slow-crack propagation rates are also dependent on the temperature and the so-called stress intensity factor K_1 (Irwin, 1968; Lawn and Wilshaw, 1975), a parameter that is proportional to the strength of the material and the length of the crack. Typical slow-crack propagation values from Anderson et al. (1977, Figure 15) lie in the range 10^{-1} to 10^2 cm s^{-1} when the local temperature at the tip of the crack is $\frac{2}{3}$ of the melting temperature for the material. The high strain energy associated with large stresses in a material at the tip of a tiny elliptical flow may transiently generate high temperatures. This thermal effect will lead to increased propagation rates, perhaps in a regenerative manner. For example, in alumina (Evans, 1974) at constant K_I, increasing T from 100°C to 1,200°C ($T/T_m = .05$ to $.60$) increases the crack velocity by six orders of magnitude.

Stress Corrosion and Magma Transport

Although it seems impossible at the present time to calculate crack velocities with any quantitative assurance, some general comments based on analogies with ceramic systems can be made (Wilkins et al., 1976).

The primary condition for crack propagation and melt ascent is that there be tension in the solid immediately in front of the crack, and that the magma rises up and follows the propagating locus of failure (Weertman, 1968, 1971, 1972). In this sense, the crack moves no faster than that limited by viscous flow of the low-density melt phase. One expects volatile-rich, low-viscosity melts to be more mobile than highly polymerized ones. The

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[5] Stress corrosion is a term sometimes used in the metallurgical literature to describe the behavior of loaded alloys under the influence of high temperatures and chemically active fluids. In glasses and ceramics, the same phenomena of growth of flaws in adverse environments is known as static fatigue (see Wilkins et al., 1976).

presence of a discrete fluid phase could also increase propagation rates by weakening the surrounding material by chemical reaction (Raleigh and Paterson, 1965). Wiederhorn (1967), for instance, has shown that increasing P_{H_2O} in a $Na_2O-CaO-SiO_2$ glass can increase crack velocities by several orders of magnitude, other factors remaining constant. An active fluid phase may explain the inferred or observed high emplacement velocities of the diatreme association, of some dike rocks, and perhaps of ash-flow tuffs.

Temperature also plays an active role in determining propagation rates in that the fracture strength of a material decreases at elevated temperature. The rates of chemical reactions between melt and the surroundings are also affected, high temperatures generally favor transport processes that are exponentially controlled,[6] such as mass diffusion, rates of chemical reactions, and viscous flow phenomena.

Slow-propagating cracks in the lithosphere, accompanied by magmatic activity, have been suggested as an alternative to the "hot-spot" hypothesis of volcanic ridge growth (McDougall, 1971, Turcotte et al., 1973, 1974). This idea may be difficult to test in that crack propagation at such low rates does not appear to generate seismic energy (Anderson et al., 1977). H. R. Shaw (this volume) explores in more detail a mechanism by shear failure (possibly aided by stress corrosion) for the transport of magma beneath Hawaii. It may be that careful location and magnitude-frequency studies of earthquake swarms in known volcanic regions will lead to some propagation-rate and energetic constraints on the migrating-fracture hypothesis for melt transfer. Hill (1977) has concisely summarized the qualitative features of a model for earthquake swarms in volcanic regions.

VISCOUS FLOW

At the outset it must be stated that the relationships and descriptions of flow phenomena that follow are not always *directly* applicable to magma flow. The philosophy adopted here is to consider *simple* models of flow and show with order-of-magnitude calculations where and when these models may be applicable. There is no single transport mechanism that may be universally applicable; however, the flow of magma is governed by the same balance of forces that describes flow in the oceans and atmosphere, convection in stars, and the flow of blood (a non-Newtonian fluid). Study of the fluid dynamics and heat-transfer properties of mobilized

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[6] For instance, the temperature dependence of the viscosity of a newtonian fluid is given approximately by $\eta = \eta_0 e^{-(E/RT)}$ where η_0, E, R and T represent the viscosity at some reference T_0, the activation energy for viscous flow, the ideal gas constant and the thermodynamic temperature.

magma is a field in its infancy (perhaps prenatal?); it is to be hoped that great progress will be made in the future.

Among the simplest transport processes one may envision for the upward flow of magma is that of pipe flow. A simple model consists of a magma-filled pipe of fixed diameter extending vertically downwards. Melt may be driven upwards in response to a number of forces. In *forced convection*, pressure gradients generated by the stress deviator (Appendix I) will set up horizontal and vertical stress gradients inducing flow along the axis of the pipe. In the case of *free convection*, density differences arising from compositional and thermal differences across the pipe create buoyancy forces. The velocity field (that is, the spatial variation of the velocity) of a fluid in pipe flow depends on the rheological[7] characteristics of the fluid, and on the balance between pressure and buoyancy forces causing flow, and the viscous forces (friction) retarding it. It may be expected that, in general, flow involves the superposition of pressure as well as buoyancy forces. In the sections that follow some details related to forced, free, and mixed convection with respect to magma transport are discussed.

FORCED CONVECTION· POISEUILLE FLOW

Imagine a pipe of diameter D extending from the surface to a depth L. The conduit is filled with magma of Newtonian viscosity η. The mean flow rate (u_m) and volumetric flow rate, Q, for isothermal, laminar flow are related to the pressure gradient $\Delta p/L$ by the Hagen-Poiseuille law whereby

$$u_m = \frac{4Q}{\pi D^2} = \frac{\Delta p}{8\eta L} R^2 \qquad (16)$$

In Equation (16) R represents the radius of an assumed circular pipe. For flow between vertical parallel plates the relation is not much different; the ideas discussed here can easily be extended to that case. The dependence of u_m on $\Delta p/L$, R and η is given in Table 1. From considerations of the magnitude of stress drops associated with earthquakes and the non-hydrostatic figure of the Earth, deviatoric stresses of perhaps 10^2 bars over lengths of the order of 1,000 km may be expected (Verhoogen, et al., 1970). Adoption of these figures leads to values of $\Delta p/L$ of order 10^{-1} bar km^{-1} Note that some of the flow rates in Table 1 are of the same order as measured slowly propagating cracks in ceramic materials.

[7] Rheology is the science of deformation and flow. Parameters such as the crystal content, degree of polymerization, temperature, pressure and amount and nature of volatiles are all important in governing the relationship between shear stress and strain rate in magma. A simple constitutive relationship is that corresponding to Newtonian flow where the magnitude of the shear stress is directly proportional to the strain rate. The constant of proportionality is η, the newtonian viscosity.

Table 1. Dependence of u_m on $\Delta p/L$, R and η calculated from Equation (17) for laminar isothermal Poiseuille flow.

Conduit Radius R (m)	Pressure Gradient $\Delta p/L$ (bar km^{-1})	Mean Velocity u_m (cm s^{-1})
	$\eta = 10$ poises	
.5	10^{-2}	3.1
	10^{-1}	31.0
	1	313.0
2.0	10^{-2}	50.0
	10^{-1}	500.0
	1	5000.0
5.0	10^{-2}	313.0
	10^{-1}	3130.0
	1	31250.0
	$\eta = 10^2$ poises	
.5	10^{-2}	0.3
	10^{-1}	3.0
	1	30.0
2.0	10^{-2}	5.0
	10^{-1}	50.0
	1	500.0
5.0	10^{-2}	31.0
	10^{-1}	313.0
	1	3130.0
	$\eta = 10^3$ poises	
.5	10^{-2}	0.03
	1	3.1
2.0	10^{-2}	0.5
	1	50.0
5.0	10^{-2}	3.1
	1	313.0

A limiting case of pressure-driven flow occurs when the pressure difference originates purely from the density difference between melt and crystalline solid. If the entire tube is filled by magma, then the pressure difference at the base of a tube of length L is found by balancing the lithostatic pressure against the magma pressure. For these conditions one computes

$$\Delta p \sim \Delta \rho\, gL. \qquad (17)$$

If $L \approx 60$ km, for example, $\Delta p \approx 2$ Kb when $\Delta \rho = .3$ gm cm^{-3} This differential pressure is sufficient to propel magma upwards at an average rate of about 4 m s^{-1} (with $\eta = 300$ poises, $R = \frac{1}{2}$ m). Simple Poiseuille flow, however, is too facile for a number of reasons. Perhaps the most limiting assumptions are: (1) isothermal flow, and (2) the presence of a long, deep fissure or pipe. The means whereby a simple long crack or pipe could open (and remain open) are not clear.

FREE CONVECTION

In pure *natural* or *free convection*, the motion is caused solely by buoyancy forces. These buoyancy forces may be of compositional or thermal origin. As an example, consider an anhydrous basalt liquid with $\rho = 2.75$ gm cm^{-3} at 1,200°C. The decrease in density as the temperature and mass fraction of dissolved H_2O increase is given by

$$-\Delta\rho = 5.2 \times 10^{-5}\theta + 1.0w_{H_2O} \qquad (18)$$

Equation (18) is based on data in Carmichael et al. (1977) and Shaw (1974) and θ is defined as

$$\theta = T - 1,473. \qquad (19)$$

Given the likely temperature gradients established across or along a narrow dike and allowing for the possibility of mass transfer of H_2O from the country rock into the magma (Shaw, 1974), local density perturbations can be significant. For instance, with $\theta = 200°K$ and $w_{H_2O} = .03$ (3 mass percent H_2O in the melt),

$$-\Delta\rho = .04. \qquad (20)$$

Although this may seem to be a small difference, it could lead to significant buoyancy forces. A rough guide to the magnitude of density-induced convection velocities is given by

$$u = \left(\frac{g\,\Delta\rho\,\kappa L}{\eta}\right)^{1/2}, \qquad (21)$$

where κ is the thermal diffusivity and L the length scale over which the density differences occur. With $L \sim 1.0$ m, $\kappa = 10^{-2}$ cm^2 s^{-1}, $\eta = 10^2$ poises and $|\Delta\rho| = .04$, velocities of the order of a few cm s^{-1} result.

MIXED CONVECTION

As indicated above, neither of the two idealized models discussed pertains very closely to the flow of magma in natural conduits. One may anticipate that most flows will be driven by *concurrent* buoyancy and pressure gradients. A large amount of experimental and theoretical effort has been invested in the study of heat and momentum transfer in mixed convection conduit flow (Goldstein, 1938; Eckert et al., 1950; Lighthill, 1953; Ostrach, 1954; Hallman, 1956; Ostrach et al., 1958; Morton, 1960; Rohsenow et al., 1962; Scheele et al., 1962; Takhar, 1968). These studies may be applied to the problems of magma flow and heat transfer to obtain some idea of the geometry and transient evolution of magmatic bodies. The flow regime exhibited in a particular case depends on the externally im-

posed thermal, kinematic, and dynamic boundary conditions.[8] The imposition of horizontal and vertical temperature gradients can greatly influence the flow configuration and lead to turbulence in the ascending fluid. In studies of thermal convection in tubes, it may be shown (Lighthill, 1953) that the product (R/L) Ra characterizes the kind of flow to be expected.[9] For example, at constant R and Ra, as R/L varies from 1 to $\frac{1}{100}$, flow patterns change from the thin boundary layer type (boundary layer not filling tube) to fully developed flow without a well-mixed isothermal core region and finally to a pattern in which a stagnant portion of fluid lies above or below the fully developed laminar boundary layer (see Figure 12 in Shaw (1965) and Figures 2, 3, and 6 in Lighthill (1953)). At very high values of Ra, turbulent flow would be expected.

ROLE OF TURBULENCE

The accurate prediction of the physical conditions necessary for turbulent magmatic flow in conduits or pluton size bodies is a difficult but important task (Appendix II). The velocity of a fluid in turbulent flow differs fundamentally from one in laminar flow. In the former case, fluctuating velocity components are superimposed on the mean (time-averaged) velocity field. These fluctuating components are capable of redistributing heat, momentum, and mass (e.g., a phenocryst population or a particular chemical species) in a highly efficient manner. Rates of transfer of these quantities are therefore expected to be higher in a turbulently convecting magma body. These predictions have been shown to be accurate in numerous experimental and theoretical studies (Deissler, 1955; Eckert et al., 1950). The unequivocal petrological demonstration that a given body was in a turbulent regime, however, is difficult to prove partly because flow regimes are markedly time dependent. In a pluton cooling by free convection, for instance, temperature differences between magma and surroundings, responsible for flow in the first place, decrease with time. The efficacy of convective mixing and transport would be expected to damp out accordingly, and all indications of turbulence, such as a random distribution of early crystallized solids, might be erased.

Although the conditions necessary for turbulent flow of an isothermal Newtonian fluid in a long, smooth tube are reasonably well known

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[8] The term flow regime is meant to refer to the qualitative geometric features of a flow. A flow pattern characterized by a well-mixed isothermal core region with thin boundary layers near the walls is an example of what may be called a boundary layer regime. The pattern of flow in which temperature and flow rates vary continuously across the body is called fully developed flow.

[9] R and L refer to the radius and length of the tube. Ra is the Rayleigh number defined by Ra $= \alpha g \Delta T R^3 / \kappa \nu$ where α, κ, ν and ΔT represent the isobaric expansivity, thermal diffusivity, kinematic viscosity and axial temperature drop respectively.

(Goldstein, 1938), the applicability of this criterion to magmatic conduit flow is of little use. The ubiquitous presence of thermal gradients in magmatic flows, coupled with the demonstration (Scheele et al., 1962), that, in non-isothermal flow, instability sets in at a much lower flow rate implies that any analysis of turbulent flow is likely to be misleading unless temperature effects are explicitly considered. Presented in Appendix II is a discussion of the magnitudes of pressure and superadiabatic temperature gradients necessary for the establishment of turbulent flow in the idealized limiting cases of forced, free, and mixed convection. The conclusion reached is that, for the magnitudes of possible vertical pressure and temperature gradients within the Earth, turbulent pipe flow may be a common phenomenon. This conclusion is especially true for low viscosity (generally basaltic) melts.

ROLE OF CONVECTION

Since numerous workers (Bartlett, 1969; Shaw, 1965, Spera, 1977) have demonstrated the likelihood of natural convection in magma chambers and ascending magmatic diapirs, little discussion is called for here.[10] The essential idea is that convection does not begin until the Rayleigh number (Ra) exceeds some critical value,[11] which depends on the imposed boundary conditions, but is usually in the neighborhood of 1,500. Ra may be thought of as the ratio of buoyancy forces favoring convection to the viscous forces retarding flow. A natural consequence of the large size of magma bodies and the physical properties of silicate melts is that magma bodies will tend to exhibit convection.

The magnitude of Ra plays an important role in the determination of the spatial and temporal evolution of convection velocities and corresponding temperatures within the convecting body. The relationship between Ra and the rates of mass and heat transport in the limiting case of boundary layer (high Ra) convection may be found in Shaw (1974), Carmichael et al. (1977) or Spera (1977). Bartlett (1969) has considered the effect of natural convection on the distribution of phenocrysts in a convecting magma chamber. He gives expressions for the variation of the particle (phenocryst) population density with height as a function of the

[10] It is conceivable that an upward concentration of volatiles (notably H_2O) or a downward concentration of phenocrysts in a magma chamber can more than compensate for an unstable density distribution resulting from heating from below, thereby inhibiting convection (see Elder, 1970; Hildreth, 1977; Wright et al., 1971).

[11] One should note that this criterion assumes newtonian behavior for magma. Suspended crystals in a silicate liquid result in a melt with a finite yield strength. The usual criterion for convection (Ra > Ra_{crit}) assumes Newtonian behavior and so may not be entirely appropriate.

Stokes terminal settling velocity and Ra. These calculations may be useful for low phenocryst contents, but the deviation from Newtonian behavior should be considered in highly polymerized melts or in melts with a significant crystallinity.

MAGMA ASCENT RATES: INFERENCES FROM XENOLITHS

The occurrence of lower-crustal or upper-mantle xenoliths in a basalt flow may be used to estimate the ascent velocity of the magma. A minimum ascent velocity is found by balancing frictional and buoyancy forces acting on a xenolith settling at its terminal velocity (u_n). The magnitude of u_n determines the particle Reynolds number defined as

$$Re_n = \frac{\rho_l D_n u_n}{\eta_l},$$ (22)

which in turn is related to the value of the drag coefficient for a spherical nodule settling through melt of Newtonian viscosity η_l.[12] Balancing frictional and buoyancy forces and solving for u_n one finds

$$u_n = \left(\frac{8 R_n \Delta \rho g}{3 C_d \rho_l}\right)^{1/2},$$ (23)

(Carmichael et al., 1977) where u_n, R_n, $\Delta\rho$, ρ_l and C_d represent the nodule settling velocity, radius, density difference between melt and xenolith, density of melt, and drag coefficient, respectively. C_d is related to the nodule settling velocity and hence to Re_n. For $Re_n < .1$ one has

$$C_d = \frac{24}{Re_n},$$ (24)

and when $Re_n > .1$

$$C_d = 18 \, Re_n^{-3/5}$$ (25)

These relations have been set out in more detail elsewhere (Carmichael et al., 1977). As an example, consider the inferred magma ascent rate based on the size of an ultramafic xenolith from the 1801 eruption of Hualalai Volcano, Hawaii. Substituting the parameters $\rho_n = 3.45 \text{ gm cm}^{-3}$, $D_n = 30 \text{ cm}$, $\rho_l = 2.8 \text{ gm cm}^{-3}$ and $\eta = 350$ poises, one computes $Re_n = 14.9$, $C_d = 3.55$ and so $u_n = 51 \text{ cm s}^{-1}$ (1.8 km hr^{-1}). The actual entrainment velocity must have been somewhat larger than the settling rate since the xenolith has obviously moved upwards.

[12] It is not necessary to assume spherical particles. The interested reader is referred to McNown and Malaika (1950) for a consideration of the effect of particle shape on settling velocity.

It is important to realize that this calculation assumes Newtonian behavior for the melt. Sparks et al. (1977) have pointed out, in fact, that a melt containing a few percent crystals by volume may behave rheologically like a Bingham plastic. Such a material has a rheological "equation of state" of the form

$$\sigma = \sigma_0 + \mu\dot{\varepsilon}, \tag{26}$$

where σ is the stress $\dot{\varepsilon}$ the strain rate, σ_0 is the yield strength and μ the coefficient of viscosity. A xenolith must be a minimum size if it is to sink through a Bingham plastic melt because of the finite yield strength of the magma. The minimum radius(r^*) is approximately given by (Timoshenko and Goodier, 1970; Sparks et al., 1977)

$$r^* = \frac{3K\sigma_0}{4\Delta\rho g}, \tag{27}$$

where K is a dimensionless constant equal to about 5.0 and σ_0 is the yield strength. There are only a few measurements of σ_0 (Shaw et al., 1968; Sparks et al., 1977); measured values are in the range of $10^2 - 10^3$ dyne cm^{-2} There is some evidence that σ_0 depends on the volume fraction of crystals (Soo, 1967) according to

$$\sigma_0 = k_1\phi^3, \tag{28}$$

where k_1, ϕ, and σ_0 represent an empirically derived constant having the dimensions of stress, the volume fraction of suspended crystals (assumed spherical) and the yield strength respectively. Use may still be made of (24) to determine settling rates if an account of the non-Newtonian properties of the melt is made. For the non-Newtonian case, the settling velocity, u_n, becomes

$$u_n = .344 \left(\frac{\Delta\rho g}{\rho_l}\right)^{5/7} \left(\frac{\rho_n}{\eta}\right)^{3/7} \left(R_n - \frac{3K\sigma_0}{4\Delta\rho g}\right)^{8/7} \tag{29}$$

when $\text{Re}_n > .1$. (The result for $\text{Re}_n < .1$ can be derived in the same way.) Notice that in (29) the yield strength enters into the balance for the settling rate. Table 5 gives some numerical examples of u_n for a variety of σ_0 values. Also indicated in Table 5 are approximate ϕ values (volume fraction crystals) consistent with Equation (28) and the few measurements of ϕ versus σ_0 for basaltic magmas. Note that a 14 cm diameter xenolith would exhibit no tendency to sink in a melt with $\sigma_0 \simeq 1,200$ dyne cm^{-2}; this might correspond to a crystallinity of about 35%. These relationships point out the need for detailed observations regarding the crystallinity of nodule-bearing (generally alkalic) basaltic flows and the abundance and size distribution of enclosed xenoliths. Further measurements concerning the dependence of σ_0 on ϕ should be undertaken, Equation (28) has not been adequately tested in silicate crystal-liquid suspensions.

Table 5. Settling rates of xenoliths in basaltic magmas. The settling rate, u_n, is computed from Equation (29). These are minimum ascent rates; for xenolith entrainment the magma velocity would have to exceed the settling rates listed here. The following parameters have been assumed constant: $\Delta\rho = .65$ gm cm^{-3}, $\rho_l = 2.8$ gm cm^{-3}, $\eta = 350$ poises and $K = 5.0$. ϕ (volume fraction solids) is computed from Equation (28) with $k_1 \approx 3 \times 10^4$ dyne cm^{-2}. The effective radius is defined as $R_n - r^*$ and r^* is given by Equation (27).

Spherical Xenolith Radius R_n (cm)	Effective Radius $R_n - r^*$ (cm)	Yield Strength σ_0 (dyne cm^{-2})	Volume Fraction Crystals ϕ	Nodule Settling Velocity u_n (cm s^{-1})
15.0	15.0	0	.0	51.0
15.0	13.2	300	.21	44.0
15.0	12.0	500	.26	39.2
15.0	9.1	1000	.32	28.7
15.0	3.2	2000	.41	8.8
7.0	7.0	0	.0	21.0
7.0	5.2	300	.21	15.0
7.0	4.0	500	.26	11.1
7.0	1.1	1000	.32	2.6
7.0	.0	1190	.34	0.0

ROLE OF VISCOUS DISSIPATION

Since the pioneering work of Gruntfest (1963, 1964) and Shaw (1969) the possible role of viscous dissipation in conduit flow and magma genesis has been discussed by a number of workers (Anderson et al., 1974; Fujii et al., 1974; Hardee et al., 1977). The essential idea behind viscous dissipation is as follows. In a material undergoing penetrative deformation, mechanical kinetic energy will be degraded through the action of viscous stresses to thermal energy (i.e., frictional heat). If the rate of viscous heat production exceeds the rate of heat removal by convection or conduction, the material will heat up, or melt, or both. Whether viscous dissipation is important depends, therefore, on the thermal and kinematic boundary conditions, the physical properties of the melt (notably the viscosity-temperature dependence) and the size of the body or conduit. According to Gruntfest (1963) the condition that shows whether viscous heat production can be balanced by heat conduction or not is identified by a critical value of dimensionless parameter that has since been termed the Gruntfest number, Gu. The value of Gu required for a runaway heating or melting episode depends on boundary conditions and the geometry. Gu has been defined as

$$\text{Gu} = \frac{a\sigma_o^2 l^2}{k\eta_0}, \tag{30}$$

where a is a parameter relating η to T (i.e., $\eta = \eta_0 e^{-a(T-T_0)}$ and σ, l, k, and η_0 represent the constant externally-imposed shear stress, the thickness of the layer undergoing shear, the thermal conductivity and the viscosity at T_0, a reference temperature, respectively. $Gu_{crit} = .88$ for plane shear at constant stress where the initial temperature is everywhere the same and equal to T_0. As an example, consider the possibility of viscous heating in a horizontal layer in the mantle of thickness l and initial Newtonian viscosity of 10^{21} poises. If a constant shear stress acts across the layer and $Gu > Gu_{crit}$ viscous heat generation cannot be balanced by heat conduction and thermal runaway will occur. Taking the thickness of the shear zone to be 10 km and with $a = 10^{-1}\,K^{-1}$, $\sigma = 10^2$ bars and $k = 3 \times 10^5\,erg\,cm^{-1}\,K^{-1}\,s^{-1}$ (Spera, 1977; Shaw, 1968; Carmichael et al., 1977), $Gu = 3.3$. Viscous heat would be generated at a rate greater than heat could be conducted out of the plane layer. Presumably, temperature would rise in the layer until T approaches T_s, the solidus temperature, at which point partial fusion will commence. Thereafter, temperature would increase less rapidly as viscous heat production supplies the energy necessary for partial fusion. After a significant portion of melt has been generated, its upward migration would alter the thermal characteristics of the system because ascending melt would very efficiently transport heat upwards, away from the zone of deformation (Feigenson et al., 1980).

For flow of magma in a conduit, Hardee et al., 1977 showed by an approximate solution to a simplified version of the conservation of energy equation that the viscous heat-production/heat-loss balance determines a critical dike width. For widths smaller than the critical value the heat loss outstrips production and the material in the pipe freezes. For basaltic systems, computed critical dike widths are of the order of 1 meter. This is generally consistent with the observed size of basalt dikes (Fujii et al., 1974; Bodvarsson et al., 1964; Mohr et al., 1976). After flow has been established, viscous-dissipation effects could be important in maintaining or accelerating the flow.

In terms of the applicability of viscous dissipation as a mechanism for concentrating volcanic heat, mention should be made of the work by Shaw (1973). Thermal-feedback viscous-dissipation theory predicts an accelerating pattern of magma production due to the action of constant viscous stresses in the magma source region. The theory also predicts a characteristic instability time, indicative of the time necessary for the culmination of a volcanic episode. The rate of eruption of Hawaiian basaltic magmas over roughly the last 10 m.y. was found to be consistent quantitatively with the constant-stress thermal-feedback theory. Whether this is coincidence or not can only be decided by further tests, either natural or laboratory, of the theory. It may be that the observed exponen-

tial increase in the volumetric eruption rate of basaltic volcanism at Hawaii (on a 5 m.y. time scale) is not related to the magma production rate at all but instead is a manifestation of an increased availability of flow paths that ultimately lead to the surface. As noted earlier, slow crack propagation rates depend exponentially on temperature, so that, as a region of mantle is heated up by magmatic heat from below, crack propagation rates would be higher and hence the likelihood of any particular batch of magma reaching the surface would be greater. In addition to the study of rates of volcanic eruption in oceanic island or island-arc provinces, a detailed analysis of the rate of production of oceanic crust along a diverging plate margin, where adequate high resolution age data are available, might put some constraints on the melt production rate and its variation in time. Episodic sea-floor spreading would result in episodic magma production rates. Of all the environments where magma is produced it is suggested that oceanic spreading zones represent the ones where access to the surface is most available.[13]

One set of experiments described by Spera (1977, Chapter IV) illustrates that a flow system subjected to a constant strain rate may exhibit periodic thermal phenomena (i.e., episodic temperature fluctuations at a fixed point in the fluid), the frequency being orders of magnitude smaller than that of the disturbing function (i.e., the constant strain rate). For the particular geometry of his experiment [cylindrical couette flow of a viscous ($\eta_0 \approx 10^3$) fluid], the period of the temperature perturbation is approximately equal to

$$\tau = 2R_o^2 \kappa^{-1}\,Ra^{-2/5}, \tag{31}$$

where R_o and κ represent the half-width of the deforming region and the thermal diffusivity ($\kappa = k/\rho C_\rho$), respectively. For these experiments Ra is given by the expression

$$Ra = \frac{4\rho\alpha g R_o^3 R_i^2 \Omega^2}{\kappa k}, \tag{32}$$

where R_i and Ω denote the radius of the rotating shaft (source of kinetic energy) and its angular velocity (rad s^{-1}), respectively. The strain rate ($\dot{\varepsilon}$) of the fluid is related to Ω and distance r from the center of the deforming region by

$$\dot{\varepsilon} = \left(\frac{2R_i^2 R_o^2}{R_o^2 - R_i^2}\right)\left(\frac{\Omega}{r^2}\right). \tag{33}$$

[13] The presumption of accessibility is based on the inferred tensional environment as indicated by fault plane solutions of earthquakes along ridges. As shown above, in tensional environments the conditions for melt intrusion do not require $p_f > p_{lith}$.

The maximum strain rate occurs at $r = R_i$, along the margins of the rotating shaft. The maximum $\dot{\varepsilon}$ is therefore given by

$$(\dot{\varepsilon})_{max} = \left(\frac{2R_o^2}{R_o^2 - R_i^2}\right)(\Omega). \tag{34}$$

When the shaft rotation rate is greater than about 1,200 rpm ($\dot{\varepsilon}_{max} = 252$ s^{-1}), a periodic temperature fluctuation at a fixed position in space is detected by a continuously recording thermocouple located there. The amplitude and frequency of the temperature fluctuation both increase as the strain rate increases. The period of the temperature fluctuation is theoretically determined by (31) and the experimental data confirm this relationship. For example, at 1,200 rpm, the theoretical period of the temperature perturbation is 745 s versus the actually measured value of 765 s. The predicted period of the temperature oscillation should be compared to the period of the mechanical driving force (i.e., rotation period of the shaft) which is given approximately by $\dot{\varepsilon}^{-1}$ and is on the order of 10^{-3} s. The striking feature that emerges from these experiments and others (e.g. Shaw, 1967, unpublished data) is the demonstration that even a relatively simple thermomechanical system may exhibit periodic thermal phenomena in response to a mechanical source of energy which is periodic itself but on a vastly different time scale. For these experiments, the ratio of the two time scales is of the order of 10^6. Although it may be entirely coincidental, it is interesting to note that the ratio of the velocities at which lithospheric plates move to typical magma ascent rates is about the same.

DIAPIR MECHANICS

Flows involving density instabilities due to density inversions have been widely discussed in the fluid dynamic and geological literature (Chandrasekhar, 1961, Ramberg, 1963, 1967, 1968a, 1969b, 1970; Parker et al., 1955). Whitehead and Luther (1975), among others, have performed scale experiments demonstrating the physical features associated with density instability. The classical Rayleigh-Taylor instability problem is the superposition of two layers of fluid; the bottom layer, being less dense than the upper layer, is inherently unstable. By means of an isothermal linear stability analysis one can compute both the distance between successive pipes and the initial growth rate of the upwardly ascending low-density phase. Marsh et al. (1974) applied this theory to explain the observed spacing between volcanic centers in some island arc terrains. They noted that, if melt forms at depths corresponding to the top of the seismic

Benioff zone beneath the volcanic chain, then the thickness of the melt layer is on the order of 10^2 m for a characteristic volcanic center wavelength of 70 km. The experiments of Whitehead et al. (1975) indicate that a growing buoyant mass begins its migration upwards when the Stokes ascent rate for the body exceeds its rate of growth. The latter quantity is proportional to the rate of production of the low density fluid.

While there is no doubt that the Rayleigh-Taylor instability theory has applications to some problems in geotectonics and geophysical fluid dynamics, its limitation to nonreactive isothermal systems makes application to magma transport studies questionable. A brief calculation illustrates this inadequacy. Assume a mostly segregated batch of magma of viscosity 10^2 poises rising through the mantle ($\eta_m = 10^{21}$ poises). Ascent rates appropriate for carrying ultramafic xenoliths, and consistent with rates of slow cracking by stress corrosion, may be conservatively estimated to be on the order of 10^{-2} cm s^{-1} The radius (R) of the assumed spherical diapir may be calculated as

$$R = \left(\frac{3u\eta_m}{g\,\Delta\rho}\right)^{1/2}, \tag{35}$$

where η_m is the viscosity (assumed Newtonian) of the mantle and $\Delta\rho$ represents the density difference between solid peridotite and the melt phase ($\Delta\rho \simeq .5$ gm cm^{-3}). Using these figures, one computes from Equation (35) that $R = 2,500$ km—clearly an unreasonable figure. In order to compute reasonable values of R,[14] η_m needs to be of order 10^{11} poises, many times smaller than the oft-quoted figure of 10^{21} poises for the mantle viscosity.

The calculation performed above fails because the model upon which it is based is too simplistic. Perhaps the most severely limiting factor is that, undoubtedly, heating of the country rock due to the motion of a hot diapir results in a lowering of the effective viscosity of the immediately adjacent mantle. In this view, a large blob of magma may be imagined to rise through the relatively cool lithosphere by softening a thin rind of wall rock, causing it to flow past the magma. The effective viscosity of the wall rock is significantly lowered in response to transient heating and partial melting. Quantitative phenomenological models based on country-rock/magma-diapir thermal interactions have been proposed for ascending alkali basalts (Spera, 1977; Carmichael et al., 1977) and andesite diapirs (Marsh, 1976a, 1976b, 1978; Marsh et al., 1978). These models enable one to predict temperature-depth trajectories of ascending batches

[14] As an example, consider the remarkable basalt flow of the 1801–1802 eruption of Hualalai Volcano on Hawaii. Its estimated flow volume of about 0.20 km^3 corresponds to an effective magma sphere of diameter 340 m.

of magma. The computed trajectories take account of most first-order processes throught to be relevant to ascent, including heat conduction and convection into country rock, partial fusion of country rock, crystallization of liquidus phases within the melt, PV work done by the expanding magma, and viscous heating. For example, a 1 km (diameter) diapir of alkali basalt, ascending at a rate of 1 cm s^{-1} from a depth of 50 km, partially fuses a rind of peridotitic country rock roughly .5 m thick to the extent of about 30%, giving effective mantle viscosities of around 10^{11} poises. The temperature loss of the diapir due to the heating effect is about 50°C. Because andesites rarely contain xenoliths, minimum ascent rates are difficult to estimate. Marsh has used ascent-rate dependent models of heat transfer between an andesitic melt and its wall rock, in conjunction with andestitic liquidus and solidus temperatures, to estimate minimum mean ascent velocities. For example, if magma is to remain molten until reaching the surface, its ascent velocity must be greater than about 3 m yr^{-1} for a body of radius 6 km and greater than 315 m yr^{-1} for a body of radius .8 km.

Although the present discussion deals primarily with the initial passage of magma through the lithosphere, repeated use of the same passageway may leave the wall rock unusually warm and therefore allow succeeding magmas to lose heat less rapidly than preceeding ones. It is very difficult to estimate the ratio of the volume melt generated at depth to the erupted volume. Presumably, in any given region, this ratio increases as time passes and the path through the lithosphere that ascending magma follows is heated up.

General Features of Diapir Ascent

Of all the mechanisms proposed to account for the generation of basaltic melts within the mantle (see Yoder, 1976, Chapter 4 for a review of some) a combination of the ones originally suggested by Yoder (1952) and Verhoogen (1954, 1973) and subsequently discussed by many others (e.g., Green and Ringwood, 1967; Ramberg, 1968c; Cawthorn, 1975; Arndt, 1977) seems the most generally applicable. The essential physics of the process is based on the adiabatic decompression of an ascending solid diapir.[15] The diapir rises because it experiences a positive buoyancy force

deriving from either compositional or thermal effects.[16] When the temperature-depth trajectory of the adiabatically ascending crystalline diapir intersects the peridotite solidus, partial fusion begins. The energy necessary for fusion comes from the heat content of the diapir, and so, if chemical equilibrium is maintained, the temperature-depth trajectory of the now crystal-liquid mush follows the steeper Clapeyron trajectory. Typical Clapeyron slopes for anhydrous peridotites are in the range 3–5°K km^{-1} whereas adiabatic slopes[17] lie in the range .3 to 7°K km^{-1}. After partial fusion has begun, there will be a tendency for melt to segregate. This tendency arises because the melt is more buoyant than the residual crystals and because of the filter pressing action of deviatoric stress (i.e., melt is kneaded out of the two-phase mush). Specific numerical calculations relating the depth at which partial fusion begins to the composition, temperature, and fraction of melt have been given by Cawthorn (1975), Ramberg (1968c), and Arndt (1977). These calculations are approximate, however, because considerable uncertainty remains about the size, kinetics of melting, role of frictional heating, and importance of convective heat loss within such rising diapirs. In an effort to illustrate more explicitly how these factors govern diapir ascent behavior the following macroscopic analysis is offered.

Thermodynamics of Diapirs

A general expression for the first law of thermodynamics (conservation of energy) as applied to a mass m in a flow system is (Lewis and Randall, 1961)

$$\Delta H + m \frac{\Delta u^2}{2} - mg \Delta Z = Q - w', \tag{36}$$

where ΔH represents the change in enthalpy ($H_{\text{final}} - H_{\text{initial}}$) of the fluid (a function of ΔP, ΔT, and f, the mass fraction of melt formed), Δu is the change in velocity and ΔZ is the difference in fluid depth ($\Delta Z = Z_f - Z_i$). Note that $-\Delta Z$ is the increase in height above some reference level and so

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[15] The assumption of adiabaticity requires that viscous heat production and heat losses by convection and conduction are negligible. Because all flows involve friction, no flow can be truly adiabatic. For a diapir to approximate adiabatic rise, a necessary (but not sufficient) condition is that its ascent time must be relatively small compared to its heat conduction characteristic time. That is, the inequality $Z/w < l^2/\kappa$ must hold. Z is the length traversed by, w the ascent rate, and l the half-width of the diapir. If a diapir of diameter 5 km ascends from a depth of 75 km, then for the flow to be considered adiabatic $w > 25$ cm yr^{-1}.

[16] See papers by Jackson and Wright (1970) and Boyd and McCallister (1976) for a discussion of the densitities of dunites, garnet websterites, garnet lherzolites and other mantle rocks.

[17] It may be shown that the adiabatic or isentropic temperature-depth relationship for a single phase is given by

$$T(z) = T_0 \exp\left[\frac{+\alpha g(Z - Z_0)}{C_p^0}\right]$$

where α and C_p^0 represent the isobaric expansivity and isobaric heat capacity of the phase and T_0 is the initial temperature (at depth Z_0).

is directly related to the potential energy increase of the mass. Q and w' on the right hand side of (36) represent the heat absorbed and energy expended in overcoming friction by the fluid, respectively. In what follows, Equation (36) is applied to the case of two-phase (e.g., crystal-liquid) flow in an attempt to evaluate the energy budget of an ascending mantle diapir undergoing partial fusion. The term fluid is used here in a completely general sense and is convenient to use when referring to the ascending diapiric material, whether it be entirely crystalline or partially fused. Note that it should be immediately obvious from simple physical considerations that the potential energy and frictional work terms are positive quantities; if the fluid moves up in the gravitational field of the earth, its potential energy increases. The expenditure of mechanical energy to overcome the retarding frictional forces will similarly always be a positive quantity. The kinetic energy will generally be positive; however, a simple order of magnitude analysis shows that the increase in kinetic energy of the fluid is generally much smaller than the other terms.[18] Because the diapir *does* move upwards to a higher potential energy state there must be compensating effects. Perhaps the most significant of these is the *decrease* in the enthalpy of the fluid. Despite the fact that partial fusion *increases* the enthalpy of the fluid (at fixed P and T), the pressure drop (ΔP) and temperature loss ($\Delta T = T_f - T_i$), incurred by the rising diapir, more than compensate for the enthalpy of melting effect. Overall, therefore, the enthalpy of the fluid decreases; it is this decrease that balances the increase in potential and kinetic energy and the production of melt.

In order to be more specific one must examine the thermal balance represented by (36) more closely. Imagine that an entirely crystalline diapir ascends without partial fusion until reaching a depth Z_i below the surface. At this depth the TZ trajectory of the diapir intersects the mantle solidus and partial fusion begins. The ambient values of P and T at the inception of partial melting are taken as P_i and T_i. Note that, in general, melt will have a tendency to segregate from the residual crystals. The energetics (not the dynamics!) of the segregation process can be modeled by allowing the melt and crystal fractions of the diapir to rise to different levels.[19] Explicitly then, the third term on the left hand side of (36) becomes

$$-mg\,\Delta Z = -m^l g(Z_f^l - Z_i) - m^{xl} g(Z_f^{xl} - Z_i). \qquad (37)$$

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[18] Even in the case of rapidly ascending kimberlites this statement appears justified. McGetchin and Ullrich (1973b) estimate for a volatile-charged kimberlite moving upwards from a depth of 100 km in a 20 m diameter pipe that $\Delta u_m \simeq 60\ \mathrm{ms}^{-1}$. The ratio of potential to kinetic energy increase becomes $2g\Delta Z/(\Delta u)^2 \simeq 600$.

[19] Because oceanic crust is 5–7 km thick, one might suggest that the melt fraction of a ridge partial fusion episode rises at least 5 to 7 km higher than the residual crystals.

Dividing by m, noting that $m^l/m = f$ and $m^l + m^{xl} = m$, and rearranging, one obtains

$$-g\,\Delta Z = g(Z_i - Z_f^{xl}) + gf(Z_f^{xl} - Z_f^l), \qquad (38)$$

where Z_f^{xl} represents the depth below the surface to which the crystals rise, Z_f^l is the depth below the surface the liquid finally attains, and Z_i is the depth at which the peridotite solidus is first intersected. The change in enthalpy of the fluid accounting for both the production of melt and the segregation process is given by (Appendix III)

$$\frac{\Delta H}{m} = c^{xl}(T_f - T_i) + (1 - \alpha T_f)g(Z_f^{xl} - Z_i)$$
$$+ f[(1 - \alpha T_f)g(Z_f^l - Z_f^{xl}) + \Delta h_{\text{fusion}}]. \qquad (39)$$

If equations (38) and (39) are combined with (36), one computes for the melt fraction, f,

$$f = \frac{Q/m - w'/m - \Delta u^2/2 + \alpha T_f g(Z_f^{xl} - Z_i) + c^{xl}(T_i - T_f)}{\alpha T_f g(Z_f^{xl} - Z_f^l) + \Delta h_{\text{fusion}}}. \qquad (40)$$

In what follows some numerical estimates of f for a number of cases are given. The following parameters are assumed constant throughout:

$$\Delta h_{\text{fusion}} = 125\ \mathrm{cal\ gm}^{-1}\ (5.2 \times 10^9\ \mathrm{erg\ gm}^{-1}),$$
$$g = 981\ \mathrm{cm\,s}^{-2}, \quad \alpha = 6 \times 10^{-5}\ \mathrm{K}^{-1},$$
$$T_f = 1{,}450\ \mathrm{K}, \quad T_i = 1{,}625\ \mathrm{K},$$
$$Z_i = 72\ \mathrm{km}, \quad c^{xl} = .3\ \mathrm{cal\ K}^{-1}\ \mathrm{gm}^{-1}\ (1.26 \times 10^7\ \mathrm{erg\ gm}^{-1}\ \mathrm{K}^{-1}).$$

It may be seen by a straightforward order-of-magnitude calculation that $\Delta u^2/2$ is generally negligible with respect to the other terms in (40). In what follows, this term is neglected.

For the case of adiabatic ($q = 0$), frictionless ($w' = 0$) flow, with no melt segregation ($Z_f^{xl} = Z_f^l = 0$), Equation (40) reduces to

$$f = \frac{\alpha T_f g(Z_f^{xl} - Z_i) + c^{xl}(T_i - T_f)}{\Delta h_{\text{fusion}}}, \qquad (41)$$

and, for example, one calculates $f = .35$ when the fluid expands from a depth of 72 km ($T_i = 1{,}350^\circ\mathrm{C}$) to the surface ($T_f = 1{,}175^\circ\mathrm{C}$).

A treatment of the general case of mantle diapirism that accounts for heat losses from the fluid and the internal friction is significantly more complicated. What follows here should be construed as only a guide to what typical orders of magnitude may be.

The energy dissipated by friction depends on the transport properties of the diapiric material, such as its viscosity and ascent rate, as well as the

geometry of the diapir itself. In a previous section it has been shown how the uncertainty regarding the "viscosity" of the mantle can lead to unreasonable results. The problem is complicated by the fact that the effective viscosity of the rising diapir is sensitive to the fraction of melt. For the admittedly unrealistic case of Poiseuille flow of a fluid with density ρ_f and viscosity (assumed Newtonian) η_f, the friction term may be written

$$\frac{w'}{m} = \frac{8\Delta Z \eta_f u_m}{\rho_f R^2} = \frac{\Delta p}{\rho_f}. \tag{42}$$

The last equality follows from the dependence of u_m on Δp for Poiseuille flow in a circular conduit. The magnitude of w'/m is seen to depend very much on the conduit size and viscosity of the fluid. Taking a typical value of Δp to be 10^2 bars gives $w'/m \simeq 3.3 \times 10^7$ erg gm^{-1}, a figure that is much smaller than the last two terms in the numerator of Equation (40). A cylindrical diapir of 1 km radius, moving upwards from a depth of 75 km at a rate of 1 cm s^{-1}, and having an effective viscosity of 10^{11} poises, gives $w'/m = 1.8 \times 10^8$ erg gm^{-1}—a figure of comparable magnitude to $\alpha T_f g(Z_f^{xl} - Z_i)$ in (40). The latter figure is probably a generous overestimate as it is unlikely that strain rates would ever reach such large values in so large a mass. The above calculations suggest that too little is known about the rheology and configuration of mantle diapirs to neglect the energy dissipated by friction in the energy budget of rising diapirs.[20]

Finally, nonadiabatically ascending diapirs may be considered. As long as the diapir is mostly crystalline, heat losses will be governed by conduction processes. These are inherently slow compared to convection rates. The time taken for heat to travel a distance 70 km, for example, may be estimated according to $t \sim l^2/\kappa = 10^{10}/10^{-2} = 3.2 \times 10^6$ yrs. A body ascending at a rate of a 5 cm yr^{-1} will travel about 200 km during this time. However, once a significant fraction of melt has formed, heat transfer between convecting magma and wall rock could become significant. In a spherical mass of segregated melt the total heat loss per gram of liquid (i.e., $-Q/m$) is given by

$$\frac{-Q}{m} = \frac{3\bar{q}Z_f^{xl}}{\rho u_m R}, \tag{43}$$

where $\bar{q}$, Z_f^{xl}, and R represent the heat flux (erg cm^{-2} s^{-1}) out of the magma body, the segregation depth, and the radius of the assumed spherical magma sphere, respectively. The heat flux $\bar{q}$ may be related

[20] Consider a mantle plume of 100 km diameter with $= 10^{21}$, $u_m = 1$ m yr^{-1} and L = 500 km; $w'/m = 1.5 \times 10^{10}$ erg gm^{-1}, a value which indicates frictional heating could be important.

to the dimensionless Nusselt number,[21] which in turn depends on the Rayleigh number, Ra. For laminar convection

$$\bar{q} = \frac{k\Delta T}{R} \, \mathrm{Nu} = \frac{1}{2} \frac{k\Delta T}{R} \, \mathrm{Ra}^{1/4}, \tag{44}$$

where

$$\mathrm{Ra} = \frac{\alpha g \Delta T R^3}{\kappa v}. \tag{45}$$

In these expressions k, ΔT and v represent the thermal conductivity ($k = \rho C_p \kappa$) of the melt, the temperature difference between the center and margins of the convecting chamber, and the kinematic viscosity of the melt (η/ρ), respectively. If Equations (44) and (45) are substituted into (43), the result

$$\frac{-Q}{m} = \frac{3k\Delta T Z_f^{xl} \, \mathrm{Ra}^{1/4}}{2\rho u_m R^2} \tag{46}$$

is obtained. Returning to the earlier expression for the fraction of melt (ignoring the frictional work expended and kinetic energy term), one finds that

$$f = \frac{(-3k\Delta T Z_f^{xl} \, \mathrm{Ra}^{1/4}/2\rho u_m R^2) + \alpha T_f g(Z_f^{xl} - Z_i) + c^{xl}(T_i - T_f)}{\alpha T_f g(Z_f^{xl} - Z_f^l) + \Delta h_{\text{fusion}}}. \tag{47}$$

With Equation (47) one may estimate the fraction of melt generated for the case previously considered, this time allowing for convective heat loss. The result of this computation is $f = .22$.[22] Recall that in the earlier case, when heat transfer was ignored, $f = .35$. Obviously, heat transfer between a hot diapir and the relatively cool lithosphere it traverses can be significant.

To summarize, it has been shown that the important factors governing the degree of partial fusion are 1) the temperature and depth where partial fusion begins, 2) the depth of "segregation" of the melt, and 3)

[21] The Nusselt number Nu may be defined as the ratio of the total heat transported out of or into a body to the heat transported if heat conduction was the only operative heat transfer process. The numerical relationship between Nu, Ra and w has been considered by numerous workers and is summarized in the text by Rohsenow and Choi (1962). See Shaw (1974), Carmichael et al. (1977) or Spera (1977) for application of these correlations to petrological problems.

[22] In addition to the figures cited earlier, we have assumed the following: $k = 3 \times 10^5$ erg cm^{-1} k^{-1} s^{-1}, $T = 10$ K, $R = 500$ m, $v = 10^2$ cm^2 s^{-1}, $\rho = 2.8$ gm cm^{-3}, $u_m = 10^{-2}$ cm s^{-1}.

heat loss by convection from the mostly liquid portion of the diapir to the surroundings. The work expended by the diapir on its surroundings (e.g., fracturing the country rock) and in overcoming internal friction is difficult to evaluate quantitatively unless something specific is said about the dynamics and geometry of flow. For viscous Newtonian flow in a pipe, frictional heating may or may not be important, depending on the internal viscosity of the diapir and the distribution of strain rates within it. The use of the term "segregation depth" is ambiguous; there should always be a tendency for chemical and thermal equilibration of melt with the surrounding rocks as long as chemical-potential gradients and temperature differences persist. On the other hand, there may be an effective value of the melt fraction for which the differential velocity between melt and crystal is so large as to render continued equilibration impossible. It may well be that, rather than a single pulse of magma rising upwards to the surface, a *series* of pulses is required. Each batch of crystallizing melt would act as a heat-transfer fluid, eventually the environment would be hot enough to permit relatively rapid rise either by ascent of a viscous blob or by swarms of propagating magma-filled cracks. Perhaps the cumulate-type xenoliths found in some alkali basalts represent such fossilized heat exchangers. In this context, the progression observed at Hawaii and some other volcanic centers, of early hypersthene-normative melts followed by increasingly alkalic melts, is intriguing. That segregation depths are less for tholeiitic melts than for alkalic melts seems compatible with the data from experimental phase-equilibria and thermodynamic calculations of magma-mantle equilibration conditions (Green, 1970, 1967; Yoder et al., 1962, Carmichael et al., 1977). Eggler, on the basis of experimental studies in the system H_2O–CO_2–basalt (Eggler, 1973; Eggler et al., 1978), has suggested an alternative model to explain the progression. He argues that early formed melts are *relatively* enriched in H_2O over CO_2. H_2O enrichment favors production of hypersthene-normative tholeiitic melts, whereas CO_2 charged melts are richer in alkalies. The demonstration by Mysen et al. (1975) of increasing CO_2 solubility in progressively more alkalic melts is consistent with this concept.

In the context of transport phenomena, Eggler's hypothesis and the increasing segregation depth model seem mutually consistent rather than inconsistent. Although early formed melts are relatively enriched in H_2O, the absolute concentration of H_2O remains below the saturation limit until very shallow levels are reached. The exsolution of H_2O does not contribute to the kinetic energy of the rising melt, therefore, until volatile saturation occurs at shallow depths. For most of its path, then, the magma moves upwards without the additional impetus of an exsolving gas. In contrast, a relatively CO_2-rich melt, because of the smaller solu-

bility of CO_2 in the liquid, is more likely to become volatile-saturated at depth. As the exsolving gas expands, it performs work upon the surrounding melt, thereby increaseing the melt ascent rate. Additionally, CO_2 in distinction to H_2O shows an exothermic heat effect when exsolving from a basic melt. The energy liberated by heat of exsolution could similarly be balanced by an increase of the kinetic energy of the melt. A corollary of this reasoning is that the more rapidly ascending alkalic melts are more capable of transporting xenoliths.

These considerations highlight the fact that evolving magmatic systems are best viewed within the context of the geologic history of a region. Geochronological and eruptive volumetric rate data may be the crucial link between the chemistry of a lava and the dynamics of ascent. An important question is posed by these considerations: what fraction of magma generated within the Earth at a given time actually rises to the surface?

Porous Media Flow

On a microscopic scale, the distribution of melt in a rising diapir will depend on the grain size of the solid phases, their crystallographic orientation, and the dihederal angle between the boundaries of melt and crystals. This latter quantity is related to the magnitude of interfacial energies between the coexisting phases. For small dihederal angles, the melt phase forms a thin intergranular film which coats all grain boundaries (Adamson, 1967).

From the partial melting experiments of Mehnert et al. (1973); Busch et al., (1974); and Arndt, (1977); and from electrical conductivity arguments (Shankland et al., 1977; Waff, 1974); it appears that silicate melts tend to wet grain boundaries. Melt will initially accumulate most rapidly along polymineralic triple and quadruple junctions; however, as the fraction of melt increases, more of the grain boundaries will be coated with melt (Arndt, 1977). Analogous effects are observed in some metal systems where even a trace of a wetting liquid is sufficient to coat all crystal boundaries (Reed-Hill, 1973; Williams et al., 1968). As the polyphase assembladge is strained, the response of the suspension will depend on the fractional area of solid-solid contacts. For small degrees of melt, some solid-solid bridges will remain and the network will possess some strength. As the temperatures above the solidus increases, so will the melt fraction:[23] Mysen et al. (1977) have shown that the variation of

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[23] Melt fraction increases at constant temperature in non-adiabatic eutectic systems. In nature, however, melting related to a continuous series of crystalline solutions might be generally expected and therefore melt fraction is a unique function of temperature (Yorder, 1976, p. 114).

the volume fraction of melt (equals ε, the porosity) with temperature is not linear. As ε increases, the area of coverage and thickness of the partial melt film increases; the fracture strength (i.e., the stress necessary to propagate liquid filled cracks) drops in the material, and the melt may be extremely mobile. On a seismic length-scale, the suspension probably behaves as a plastic material with a finite yield strength (Aki et al., 1978), preventing crystals from settling out. If pressure gradients are present, however, there will be a velocity differential between melt and crystals. The flow process could then be envisioned as a form of two-phase porous-media mixed convection. An initially heterogeneous melt distribution (perhaps resulting from an inherited metamorphric fabric) will create a spatially complicated permeability field. Highly permeable zones, favourably oriented with respect to the principal stresses, might serve as high-conductivity pathways for flow. This behavior is consistent with the ideas of Shaw (1969) regarding the role of pervasive shear strains in melt segregation, and with the recent work of Turcotte et al. (1978) pertaining to the macrophysics of melt segregation. The basic idea is that melt will flow to regions of low stress at rates greater than the surrounding deformable crystal network.

Order-of-magnitude estimates of melt flow rates may be obtained from a form of the momentum equation consistent with two-phase porous-media flow (Straus et al., 1978; Ribando et al., 1976; Turcotte et al., 1978). For low Re flow in a porous medium the Z-component (vertical) of the equation of motion is

$$-\frac{\partial p}{\partial Z} - \frac{\eta}{B} u + \Delta \rho \hat{g} = 0, \tag{48}$$

where B, $\Delta\rho$, η, u and p represent the permeability, density difference between melt and crystals, viscosity of the melt, vertical flow rate and the pressure, respectively. Defining

$$p = p_{\text{lith}} + p', \tag{49}$$

and noting that

$$p_{\text{lith}} = \bar{\rho} g Z, \tag{50}$$

Equation (48) may be written in the form

$$u = -\frac{B}{\eta} \frac{\partial p^*}{\partial Z}, \tag{51}$$

where $\partial p^*/\partial Z$ is defined according to $\partial p^*/\partial Z = \partial p'/\partial Z + \Delta\rho\, g$. The rate of flow is seen to be directly proportional to the pressure gradient driving flow, the density difference, and the permeability B; and inversely pro-

portional to the dynamic viscosity.[24] Equation (48) represents a balance between the viscous forces on a moving fluid parcel, due to fluid-grain as well as fluid-fluid friction, and the pressure gradient-buoyancy forces causing flow. Pressure gradients are intimately related to the regional pattern of tectonic stresses. Consequently, one expects different segregation rates in different tectonic environments, independent of the density difference between melt and crystals.

To compute melt velocities from Equation (51), an estimate of B must be made. The dependence of the permeability (B) on the porosity (equivalent here to the volume fraction of melt) is critical, because it is the permeability that determines the differential velocity of melt and crystals. To this author's knowledge, permeabilities of silicate mushes have not directly been determined. There is, however, a wealth of information regarding flow through packed beds and consolidated media (Gonten et al., 1967; Wyllie et al., 1950; Carman, 1956; Scheidegger, 1957; Brace et al., 1968, Brace, 1977; Bear, 1972). Although it should be clearly understood that segregation of melt from a deformable crystal mush may be mechanically different from flow through a rigid porous solid, the functional form for porosity versus permeability, suggested by the studies cited above, is hesitantly adopted for the purpose of obtaining order-of-magnitude estimates of the segregation rate (i.e., the differential velocity between crystals and liquid). A first approximation for the dependence of the permeability on the volume melt fraction (porosity) is taken as

$$B = \frac{a^2 \varepsilon^3}{9k(1-\varepsilon)^2}. \tag{52}$$

The dimensionless constant k, which is related to the tortuosity, is equal to about 5.0 (Carman, 1956), and a is the radius of the solid interlocking (assumed spherical) grains. In a recent paper Brace (1977) has suggested a somewhat more general permeability-porosity relationship; however, the uncertainty regarding the size, shape and distribution of the crystal-liquid network does not warrant a more refined approximation. Table 6 is a tabulation of calculated permeabilities assuming some typical porosity and grain size values.

Combining (51) and (52), one obtains an explicit relationship between ε and the flow velocity·

$$u = \frac{a^2 \varepsilon^3}{45\eta(1-\varepsilon)^2}\left(\frac{\Delta p^*}{L}\right). \tag{53}$$

Empirically it is noted that (53) breaks down for $\varepsilon > 75$; it applies only to fully viscous flow at small values of the Reynolds number. It is also

[24] The reader should note that thermal buoyancy forces are ignored in Equation (48).

Table 6. Permeabilities and segregation velocities for melt-crystal suspensions. The permeability, B, is computed from Equation (53) and u from (54). Constant values for k (see Equation (53)) and η have been taken as 5 and 10 poises respectively. Velocities are calculated assuming $\Delta p^*/L = 100$ bars/1,000 km. Figures in parentheses were computed assuming $\Delta p^*/L = 100$ bars/10 km. To convert from volume fraction of melt (ε) to mass fraction (f), the relation $f = \rho_l \varepsilon/\rho_s + \varepsilon(\rho_l - \rho_s)$, which reduces to $f \simeq \rho_l/\rho$ at small ($\varepsilon < .20$) porosities, is useful.

Porosity ε	Permeability B (cm^2)	Flow Velocity u $(cm\ s^{-1})$	
crystal diameter = .1 cm			
.01	5.7×10^{-11}	5.7×10^{-12}	(5.7×10^{-10})
.05	7.7×10^{-9}	7.7×10^{-10}	(7.7×10^{-8})
.10	6.9×10^{-8}	6.9×10^{-9}	(6.9×10^{-7})
.25	1.5×10^{-6}	1.5×10^{-7}	(1.5×10^{-5})
.35	5.6×10^{-6}	5.6×10^{-7}	(5.6×10^{-5})
.50	2.8×10^{-5}	2.8×10^{-6}	(2.8×10^{-4})
.70	2.1×10^{-4}	2.1×10^{-5}	(2.1×10^{-3})
crystal diameter = .2 cm			
.01	2.3×10^{-10}	2.3×10^{-11}	(2.3×10^{-9})
.05	3.2×10^{-8}	3.2×10^{-9}	(3.2×10^{-7})
.10	2.7×10^{-7}	2.7×10^{-8}	(2.7×10^{-6})
.25	8.2×10^{-6}	8.2×10^{-7}	(8.2×10^{-5})
.35	3.5×10^{-5}	3.5×10^{-6}	(3.5×10^{-4})
.50	2.2×10^{-4}	2.2×10^{-5}	(2.2×10^{-3})
.70	8.5×10^{-4}	8.5×10^{-5}	(8.5×10^{-3})
crystal diameter = .5 cm			
.01	1.4×10^{-10}	1.4×10^{-11}	(1.4×10^{-9})
.05	2.0×10^{-7}	2.0×10^{-8}	(2.0×10^{-6})
.10	1.7×10^{-6}	1.7×10^{-7}	(1.7×10^{-5})
.25	5.1×10^{-5}	5.1×10^{-6}	(5.1×10^{-4})
.35	2.2×10^{-4}	2.2×10^{-5}	(2.2×10^{-3})
.50	1.4×10^{-3}	1.4×10^{-4}	(1.4×10^{-2})
.70	5.3×10^{-3}	5.3×10^{-2}	(5.3)

important to remember that (53) applies most closely to fluid flow through a rigid porous material. Although Equation (52) is clearly an approximation, previous studies (Bear, 1972; Turcotte et al., 1978; Sleep, 1974) have shown that more complex models lead to expressions with different constants of proportionality in (52), but the same functional dependence of B on ε. Examination of Table 6 shows that permeabilities and hence segregation rates are strongly dependent on porosities (volume fraction of melt). For instance, an increase of ε by a factor of two results in flow rates twenty times larger. The porosity is unlikely to remain uniform throughout a partial melt zone, especially if inhomogeneities in the melt distribution are inherited from a subsolidus tectonite fabric. An

inhomogeneous porosity field can lead to a regenerative melt segregation effect because of the non-linear dependence of permeability on porosity. In those zones where ε is initially high, B will be large. In these regions, segregation velocities are higher, and, therefore, melt will accumulate at ever-increasing rates in low-stress regions. Tabulated in Table 6 are segregation velocities for a low-viscosity melt moving under the influence of small but finite pressure gradient. The non-linear dependence of u on ε responsible for the large increase in u as ε increases is evident from the values listed in the table. These observations are consistent with experimental evidence (see Scheidegger, 1957) that supports the notion that the permeability is inversely proportional to the effective stress ($\sigma_{eff} = \sigma_{lith} - p_f$); as $\sigma_{eff} \to 0$, B increases dramatically, and, of course, so will flow rates.

CONCLUDING REMARKS: IMPORTANCE OF RHEOLOGY

It should be clear from the foregoing considerations that, of all the factors governing the rate and mechanism of magma transport, it is the rheological properties of both the melt phase and its surroundings that are most critical.

Viscosities of silicate melts can be estimated from their chemical composition, temperature, and pressure using empirical correlations (Bottinga et al., 1972; Shaw, 1972; Murase et al., 1973; Kushiro, 1976; Scarfe, 1973; Kushiro et al., 1976). The validity of these estimates has been established only at super-liquidus temperatures, where essentially Newtonian behavior is found. It has been experimentally demonstrated that the presence of gas bubbles or suspended crystals cause some melts to behave as non-Newtonian fluids[25] (Shaw et al., 1968; Sparks et al., 1977; Pinkerton et al., 1978; Murase et al., 1973). Non-Newtonian behavior is relevant to many petrologic processes including magma ascent, heat transfer by free and forced convection in magma diapirs or magma chambers, and the settling of phenocrysts and xenoliths.

There is an extensive literature concerning the rheology of hot subsolidus silicate materials (Goetze et al., 1972, Goetze et al., 1973; Green et al., 1972, Raleigh, 1968; Stocker et al., 1973; Murrell et al., 1976). Depending on the effective stress, a variety of responses are indicated. Generally speaking, ductile behavior (plastic or viscous) is observed at

[25] Non-Newtonian behavior has been recognized in some single phase polymeric fluids due to effects related to the structure of the liquid itself (Spera, 1977). High-silica melts above liquidus temperatures may exhibit similar behavior.

high pressure and temperatures and small deviatoric stresses; olivine at 1,600°C, for instance, readily flows under small (~10 bar) deviatoric stresses (see Appendix I). Low temperatures and small effective stresses favor brittle failure. Note that, even at depths where the lithostatic pressure is large, brittle failure may ensue, provided a pressurized fluid phase is present (i.e., the effective stress is small). The effective stress concept as outlined in previous sections is extremely important in terms of magma transport, in this author's opinion. One should note that the magma-filled propagating crack and viscous diapir hypotheses are not mutually exclusive. As shown above, critical tensile stresses necessary for the propagation of cracks are smallest in materials with reduced rigidities.[26] One might expect, therefore, that cracks could most easily nucleate and grow around the partially-fused rind surrounding a viscous (mostly-liquid) blob of ascending melt. Studies of fluid-flow in non-deformable porous media indicate that fluid permeabilities increase enormously as effective stresses approaches zero. If this relationship holds true in deformable silicate mushes, then melt may be mobile even in regions with small amounts of partial melt. More likely, however, mushes behave as Bingham fluids possessing a definite yield strength. A Bingham rheology would imply that some threshold volume fraction of melt exists before melt segregation rates become large enough to preclude crystal-liquid equilibrium in the source.

Finally, in closing, it must be pointed out that many topics relevant to magma transport have been neglected in this report. Aspects such as flow differentiation (Bhattacharji, 1964; Bhattacharji et al., 1967; Komar, 1972), the formation of volcanic vents, the origin and development of compositional zoning in some magma chambers, and the density relationships between depleted and undepleted peridotites (Boyd and McCallister, 1978, Jackson et al., 1970) are just a few of the many important problems that await a clearer understanding. However, it is hoped that the techniques and approaches described in the previous sections will be of some general interest and validity to igneous petrologists and volcanologists actively pursuing the "petrological inverse problem"

SUMMARY

The main aspects of this paper are summarized below in sequential order roughly corresponding to the route magma follows from source to surface.

[26] The rigidity, μ, depends on density and Shear-wave velocity according to $\mu = \rho V_s^2$.

MELT SEGREGATION

1. The rate of melt segregation from a crystal mush is governed by the rheology of the suspension and the balance among buoyancy, pressure and viscous forces.
2. Buoyancy forces may be estimated reasonably well, using density, expansivity, and compressibility data for minerals and silicate liquids. Deviatoric stress gradients can be estimated to within perhaps an order of magnitude and are dependent upon the local tectonic environment of the segregation region. Viscous forces which act to impede melt segregation depend critically upon the rheological response of the crystal-liquid system to local conditions. Viscous forces can only be poorly estimated but are probably inversely porportional to the permeability and directly proportional to the melt viscosity. Permeabilities have not (to this author's knowledge) been measured in melt-crystal systems; empirical correlations for rigid porous media suggest a non-linear, monotonically increasing dependence of permeability on volume fraction of melt. As an example, the differential velocity (segregation rate) between crystals and liquid in a mush consisting of 30% melt is 6,500 times larger than in a mush with 2% melt.

MAGMA ASCENT RATES AND VISCOUS FLOW

1. Macroscopic energy budget calculations show that the extent of partial fusion achieved in a rising diapir depends on (1) the slope and 1 bar temperature intercept of the peridotite solidus, (2) the temperature and depth where partial fusion begins, (3) the "depth of segregation" (i.e., the depth at which the differential velocity between crystals and melt is so large as to render continued crystal-liquid equilibration impossible, (4) amount of convective heat loss between diapir and lithosphere, and (5) extent of viscous heating due to frictional processes within the diapir. Nonadiabatic heat loss effects and frictional heating cannot a priori be considered to be insignificant and depend strongly on the shape and size of the magmatic body.
2. Ascent rates for nodule-bearing alkalic magmas may be computed based on a buoyancy versus viscous force balance for Newtonian or Bingham-plastic melts. Typical values are of the order of a few cm s^{-1} and, interestingly, are largely consistent with both laboratory experiments and theoretical predictions of the rate of slow crack propagation by static fatigue or stress corrosion.
3. Viscous dissipation has been suggested as a mechanism operating in melt-source regions by which mechanical energy is degraded to thermal energy and provides heat needed for partial fusion. An experiment

illustrating the development of periodic temperature fluctuations at a fixed point in a continuously sheared non-Newtonian temperature-dependent viscosity fluid was described. It is concluded that even simple thermomechanical systems involving viscous dissipation may exhibit thermal periodicities of a vastly different time-scale than the mechanical periodicities driving the system.

4. Relevant factors governing the viscous flow of magmatic fluids in dikes, pipes, diapirs, cracks, and other types of magma conduits include the following: ambient pressure gradients, magma rheology, buoyancy forces due to temperature, compositional and crystal-liquid effects, and the magnitudes of horizontal and vertical temperature gradients surrounding the magma body. Melt flowing in long conduits (>1 km) is probably in turbulent flow (at liquidus temperatures) assuming likely values for temperature and pressure gradients. A pressure gradient of 1 bar/10 km can maintain magma flow rates of several cm s^{-1} ($\eta = 500$ poises) in a 5 m diameter dike. A few percent H_2O in an otherwise anhydrous melt can generate buoyancy forces commensurate with convection velocities of a few cm s^{-1} in the melt.

MAGMA ASCENT BY CRACK PROPAGATION

1. Effective principal stress deviators of several tens to several hundreds of bars are sufficient for the initiation of brittle failure and the propagation of cracks within the crust and upper mantle. Modification of the Griffith theory of fracture in brittle solids, accounting for time-dependent and environment-dependent fracture behavior (i.e., static fatigue), indicates that the mode of failure associated with slow-cracking can occur at stresses significantly below those predicted by classical Griffith theory. Slow-crack propagation rates in ceramic materials lie in the range 10^{-1} to 10^{-2} cm s^{-1} for local crack-tip temperatures equal to about $\frac{2}{3}$ of the melting temperature of the material. Experimentally determined slow-cracking rates in ceramic materials are comparable to earthquake swarm migration rates in some volcanic regions such a Iceland and Hawaii.

BEHAVIOR OF FLUIDIZED MAGMA

1. Virtually all melts contain some juvenile volatiles. Estimates of the volatile contents (mostly H_2O and CO_2) of tholeiitic, alkali-basaltic, andestic and rhyolitic melts range from .5 to 4 wt %, the more siliceous melts being the most volatile enriched. If account is taken of likely magmatic temperatures, solubility data may be used to estimate depths at which the development of fluidized two-phase flow may commence due to volatile saturation. Critical fluidization velocities may be cal-

culated by balancing hydrodynamic and lithostatic pressure forces. Characteristic values lie between hundreds to tens of m s^{-1} for gas-dominated systems (e.g., kimberlites) to several cm s^{-1} for melt-xenolith systems.

ACKNOWLEDGMENTS

APPENDIX I

SOME BASIC CONCEPTS OF STRESS THEORY

There are many excellent accounts of continuum mechanics, the theory of elasticity (Mase, 1970; Bridgman, 1964; Nadai, 1963, Jaeger, 1962) and the application of these theories to the deformation associated with magmatic intrusions (Murrell, 1965; Ode, 1957 and 1960; Roberts, 1970; Pollard et al., 1973, 1975, 1976a, 1976b). The discussion here is restricted to some general aspects of stress states in the Earth and emphasizes those topics that are more immediately applicable to the transport of magma.

Nine stress components are necessary to define the state of stress at a point in a body. The equilibrium of an arbitrary volume of a continuous material acted on by surface and body forces requires that the resulting force and moment acting on the volume be zero. The equilibrium condition implies that the stress tensor $\underline{\underline{\Sigma}}$ at a point is symmetric. The stress tensor may, therefore, be written in matrix form as

$$\underline{\underline{\Sigma}} = \begin{pmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{pmatrix} \tag{A1}$$

In (A1) the σ_i and τ_{ij} represent the normal and shearing stresses respectively. Because $\underline{\underline{\Sigma}}$ is a symmetric tensor, it may be diagonalized by a suitable coordinate transformation. The diagonalization procedure, which can be carried out on any real symmetric tensor, determines the directions of a set of orthogonal coordinate axes such that all off-diagonal terms of the stress tensor vanish. These axes are called principal axes, and they are

associated with the three mutually perpendicular principal stress planes. Across these planes only normal (compression or tensional) stresses act, and all tangential stresses vanish. The principal stresses are often denoted by the symbols σ_I, σ_{II} and σ_{III} and are called the maximum, intermediate and minimum principal normal stresses. Obviously they act in the direction of the principal axes. By convention $\sigma_I > \sigma_{II} > \sigma_{III}$. If it is assumed that the Earth's surface is a flat, stress-free horizontal plane, one of the principal stress directions is coincident with the vertical.

The magnitudes of the principal stresses may be found by solving the determinant formed by subtraction of σ from each of the diagonal terms of the stress tensor. The roots of the following cubic equation determine the magnitudes and directions of the principal stresses:

$$\sigma^3 - I_\Sigma \sigma^2 + II_\Sigma \sigma - III_\Sigma = 0, \tag{A2}$$

where

$$I_\Sigma = \sigma_1 + \sigma_2 + \sigma_3 \tag{A3}$$

$$II_\Sigma = \tfrac{1}{2}(\sigma_i \sigma_j - \tau_{ij}\tau_{ij}) \tag{A4}$$

$$III_\Sigma = \det \underline{\underline{\Sigma}}. \tag{A5}$$

I_Σ, II_Σ and III_Σ are known as the first, second and third stress invariants.

A convenient decomposition of the stress tensor $\underline{\underline{\Sigma}}$ is into two parts. This decomposition is effected by subtraction of the spherically symmetric mean normal stress tensor from $\underline{\underline{\Sigma}}$. The mean normal stress is defined by the relation

$$\sigma_0 = p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{A6}$$

where σ_0 corresponds to the spherically symmetric "hydrostatic" pressure (p) of hydrodynamics. An approximation often made in petrology is that p is equal to the pressure due to the superincumbent load so that

$$p = p_{\text{lith}} = \bar{\rho}gz \tag{A7}$$

If a fluid phase such as melt is present, mechanical equilibrium requires

$$p = p_{\text{lith}} = p_f = \bar{\rho}gz, \tag{A8}$$

where $\bar{\rho}$ is the average density and z the thickness of the super-incumbent pile. Generally speaking, small deviations of p_f from p_{lith} are sufficient to transport melt at considerable rates (cm s^{-1}) and distances (1–100 km).

The remainder of the subtraction of the spherically symmetric hydrostatic pressure $\underline{\underline{P}}$ tensor denoted by

$$\underline{\underline{P}} = \begin{pmatrix} p & o & o \\ o & p & o \\ o & o & p \end{pmatrix} \tag{A9}$$

from $\underline{\underline{\Sigma}}$ is called the stress deviator, because it is a measure of the deviation of the stress state from a simple isotropic one. In matrix notation this decomposition is written as follows:

$$\begin{pmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{pmatrix} = \begin{pmatrix} p & o & o \\ o & p & o \\ o & o & p \end{pmatrix}$$

$$+ \begin{pmatrix} \tfrac{1}{3}(2\sigma_1 - \sigma_2 - \sigma_3) & \tau_{12} & \tau_{13} \\ \tau_{12} & \tfrac{1}{3}(2\sigma_2 - \sigma_1 - \sigma_3) & \tau_{23} \\ \tau_{13} & \tau_{23} & \tfrac{1}{3}(2\sigma_3 - \sigma_1 - \sigma_2) \end{pmatrix} \tag{A10}$$

When the stress tensor $\underline{\underline{\Sigma}}$ has been resolved by the diagonalization process into the set of principal stresses, then the decomposition into hydrostatic and deviatoric parts is easily accomplished. That is,

$$\begin{pmatrix} \sigma_I & o & o \\ o & \sigma_{II} & o \\ o & o & \sigma_{III} \end{pmatrix} = \begin{pmatrix} p & o & o \\ o & p & o \\ o & o & p \end{pmatrix}$$

$$+ \begin{pmatrix} 2\sigma_I - \sigma_{II} - \sigma_{III} & o & o \\ o & 2\sigma_{II} - \sigma_I - \sigma_{III} & o \\ o & o & 2\sigma_{III} - \sigma_I - \sigma_{II} \end{pmatrix}, \tag{A11}$$

where now

$$p = \sigma_o = \frac{\sigma_I + \sigma_{II} + \sigma_{III}}{3}. \tag{A12}$$

The importance of the decomposition of a stress field into its deviatoric and isotropic parts is founded on the experimental observation that deviatoric stresses alone are responsible for permanent deformation by flow or fracture in rocks. Whereas the isotropic part of the stress field does not induce any permanent deformation, it does influence the mechanism by which the material may be permanently deformed. At high temperatures and confining pressures, the release of stresses by a viscous or plastic flow process is more likely than by a brittle release. If in addition to high confining pressures, however, a fluid phase at a pressure close to the lithostatic value exists, then catastrophic failure by the propagation of cracks (i.e., brittle failure) is possible (See Shaw, this volume).

ROLE OF FLUID PRESSURE

Anderson (1951) and Hubbert and Rubey (1959) were among the first to develop and apply the concept of effective stress to failure in rocks. The essential idea can be grasped if one considers the walls of a cavity in an

elastic body. If the cavity remains empty, the state of stress at a point along the boundary of the crack is that given by the principal stresses, σ_I, σ_{II} and σ_{III} at that point. If, however, the crack is filled with a fluid (e.g., a silicate liquid) at pressure p_f, then the state of stress at that point is *mechanically equivalent* to one in which the stresses $\sigma_I - p_f$, $\sigma_{II} - p_f$ and $\sigma_{III} - p_f$ act on the external surface of an *empty* crack of the same form. From (A11) one notes that the effective stress tensor at a point in a material under a fluid pressure p_f is simply the stress deviator. That is

$$\begin{pmatrix} \sigma_I - p_f & o & o \\ o & \sigma_{II} - p_f & o \\ o & o & \sigma_{III} - p_f \end{pmatrix} = \begin{pmatrix} 2\sigma_I - \sigma_{II} - \sigma_{III} & o & o \\ o & 2\sigma_{II} - \sigma_I - \sigma_{III} & o \\ o & o & 2\sigma_{III} - \sigma_I - \sigma_{II} \end{pmatrix}$$

(A13)

As pointed out in the text, it is the effective stresses that are important in establishing the criteria of failure in a material. One concludes that failure by brittle fracture is possible even at great depths if a fluid phase is present (e.g., in regions of partial fusion). *In situ* stress determinations of σ_I and σ_{III} within the crust show that non-zero deviatoric stresses are a common feature (Haimson, 1977). In the source regions of magmas, the presence of earthquakes probably indicates that differences in the principal stresses are being relieved (Klein et al., 1977).

APPENDIX II

Onset of Turbulence: Isothermal Pipe Flow

For isothermal flow in smooth pipes, turbulence sets in when the Reynolds number, defined as

$$Re = \frac{2Ru_m}{v},$$

(B1)

exceeds about 2,100 (Goldstein, 1938). In terms of Poiseuille flow, because the axial pressure gradient ($\Delta p/L$) is related to the mean velocity, the condition for turbulence may be written

$$\left(\frac{\Delta p}{L}\right) > \frac{4\rho v^2}{R^3} Re_{crit}.$$

(B2)

Table II presents values of $\Delta p/L$ necessary for turbulent flow as a function of v and R. The velocity of the fluid at the inception of turbulence is also given. For a melt with $\eta = 300$ poise, a 5 m wide dike would be in turbulent flow with an axial pressure gradient of .2 bar km^{-1}. The

Table 2. Critical pressure gradients and corresponding mean vertical velocities required for turbulence in circular Poiseuille magma flow under isothermal conditions. Re_{crit} is taken as 2,100 and ρ is assumed equal 3.0 gm cm^{-3}.

Radius of Pipe R (m)	Critical Pressure Gradient $\Delta p/L$ (bar km^{-1})	Critical Vertical Mean Velocity u_m^{crit} cm/s
kinematic viscosity = 10^2 cm^2 s^{-1}		
.5	202.0	2104
1.0	25.0	1041
2.0	3.1	517
5.0	0.2	208
10.0	2.5×10^{-2}	104
15.0	7.4×10^{-3}	70
kinematic viscosity = 10^3 cm^2 s^{-1}		
2.0	315.0	3.3×10^4
5.0	20.0	2100.0
10.0	2.5	100.0
15.0	.74	700.0
20.0	.3	5×10^2

order of deviatoric stress gradients in the Earth may average around 100 bars/1,000 km and so for $\eta = 300$, turbulence is expected if $R > 5$ m. At higher viscosities, pipes of larger diameter are necessary.

Onset of Turbulence: Free and Mixed Flows

Recognizing the fact that isothermal flow is highly unlikely, one may consider the conditions for turbulence in naturally convecting circular pipes. Lighthill (1953) theoretically predicted the conditions for the onset of turbulence in such systems. His predictions have been borne out for the most part by the experimental work of Lockwood and Martin (1964). The onset of fully mixed turbulent flow depends on the aspect ratio of the tube (R/L) as well as the Rayleigh number. An appropriate criterion for instability is that $[(R/L)^3 Ra]$ be greater than about 10^4.[27] The term β represents the superadiabatic temperature gradient along the vertical walls of the conduit and is equal to

$$\beta = \frac{\Delta T - (\Delta T)s}{L},$$

(B3)

[27] This criterion is a general one; the precise value depends on the boundary conditions and on how one chooses to define turbulence (see discussion in Lipps. 1971).

where ΔT is the overall temperature difference between top and bottom, $(\Delta T)_s$ the corresponding adiabatic (constant entropy) temperature difference, and L the length of the magma-filled conduit. Ra is the Rayleigh number defined as

$$\text{Ra} = \frac{\alpha g \beta L^4}{\kappa \upsilon}, \tag{B4}$$

where α, g, κ, υ represent the isobaric expansivity, acceleration due to gravity, thermal diffusivity, and kinematic viscosity of the melt, respectively. Typical values of α and κ for silicate melts are $\alpha = 5 \times 10^{-5}\,K^{-1}$ and $\kappa = 10^{-2}\,cm^2\,s^{-1}$ (Carmichael et al., 1977; Shaw, 1974). Table 3 presents a tabulation of minimum superadiabatic temperature gradients needed for turbulence for a variety of dike widths and magma viscosities.

Table 3. Critical values of the superadiabatic temperature gradient for turbulence by free convection in a circular tube. Ra_{crit} is taken as 10^4. Assumed constant throughout are $\kappa = 10^{-2}\,cm^2\,s^{-1}$ $\alpha = 5 \times 10^{-5}$, $g = 10^3\,cm\,s^{-2}$ and $L = 10$ km.

Radius of Pipe R (m)	Critical Superadiabatic Temperature Gradient $(K\ km^{-1})$
kinematic viscosity $\upsilon = 10^2\ cm^2\ s^{-1}$	
.5	.16
1.0	.02
$\upsilon = 10^3\ cm^2\ s^{-1}$	
.5	1.6
1.0	.2
3.0	7.4×10^{-3}
$\upsilon = 10^4\ cm^2\ s^{-1}$	
.5	16.0
1.0	2.0
2.0	7.4×10^{-2}
$\upsilon = 10^5\ cm^2\ s^{-1}$	
.5	160
1.0	20
3.0	7.4×10^{-1}

An examination of the approximate figures in this table indicates that turbulence could be expected for rather small temperature gradients. Again, note that for a given viscosity, increasing R tends to make the flow more prone towards instability. For example, with $\upsilon = 10^3\,cm^2\,s^{-1}$ and $R = 1$ m, β must exceed $.2°K\,km^{-1}$ for a tube 10 km long for turbulent flow to be established. If temperature gradients such as those thought to exist in the upper parts of lithospheric plates are present (of the order $10°K\,km^{-1}$), one anticipates turbulent flow to be more the rule than the exception. Also note that increasing υ tends to damp out turbulent flow. In a 1 m radius tube 10 km long, a $10°K\,km^{-1}$ superadiabatic temperature gradient is not sufficient to generate turbulence if $\upsilon > 10^{4.5}\,cm^2\,s^{-1}$, approximately.

The general case of "mixed" flow is probably more applicable to magma transport than either of the cases discussed above. That is, one expects that pressure as well as buoyancy forces play a significant role in magma transport. Scheele and Hanratty (1962) have proposed a criterion for the transition to unsteady (turbulent) behavior in flows where buoyancy *and* pressure forces are important. The presence of both horizontal and vertical temperature gradients enhances the possibility of turbulence. Because temperature gradients across the width of the conduit are present, there is heat transfer from the margins of the magma filled conduit to the surroundings. If heat losses from the margin of the tube are large enough, sufficiently large negative buoyancy forces can be established there. These may counteract the pressure driving forces (always acting to allow magma ascent) causing an inflexion in the velocity profile. From the experiments of Scheele and Hanratty (1962), an appropriate criterion of instability is when the parameter Ra/Pe exceeds a critical value of about 53. Pe is the Peclet number and represents the nondimensional ratio of the heat transported by convection currents to that by molecular conduction. In dimensional form, the criterion relating the magnitude of $\Delta p/L$ to the critical superadiabatic temperature gradient, β_{crit}, is

$$\beta_{crit} = \frac{1}{8\rho g \alpha R}\left(\frac{\Delta p}{L}\right)\left(\frac{\text{Ra}}{\text{Pe}}\right)_{crit} \tag{B5}$$

Collected in Table 4 are typical values of β and $\Delta p/L$ for a variety of conditions. For example, if β exceeds $.44°K\,km^{-1}$ along the walls of a vertical conduit of diameter 20 m and the impressed pressure gradient is 1 bar km^{-1}, the flow will be turbulent. Large width conduits are more prone to turbulence, other factors remaining constant. If pressure gradients are are on the order of 1 bar km^{-1}, a superadiabatic temperature gradient of $5°K\,km^{-1}$ is not great enough to cause turbulent flow in a 1 m diameter conduit.

Table 4. Critical values of β the superadiabatic temperature gradient as a function of conduit size and the vertical pressure gradient. Assumed constant have been $\rho = 3.0$ gm cm^{-3} $g = 10^3$ cm s^{-2} and $\alpha = 5 \times 10^{-5}$ K^{-1}. (Ra/Pe)$_{crit}$ is taken as 53.0.

Radius R (m)	Pressure Gradient $\Delta p/L$ (bar km^{-1})	Superadiabatic Temperature Gradient β_{crit} (K km^{-1})
.5	10^{-2}	.09
	10^{-1}	.9
	1	9.0
1	10^{-2}	.044
	10^{-1}	.44
	1	4.41
	10	44.1
3	10^{-2}	.015
	10^{-1}	.15
	1	1.5
	10	15.0
10	10^{-2}	4.4×10^{-3}
	10^{-1}	4.4×10^{-2}
	1	.44
	10	4.4

The general conclusion to be drawn is that, for the simplified cases discussed above, turbulence in tube flow may be expected to be common. The temperature and pressure gradients expected within the lithosphere (of order $10°$K km^{-1} and 10^{-1} bar km^{-1} respectively) indicate turbulence even for relatively narrow conduits.

APPENDIX III

The change in enthalpy is given by

$$\Delta H = H_{final} - H_{initial} \tag{C1}$$

$$\Delta H = m^{xl}h^{xl}_{T_f,P_f} + m^l h^l_{T_f,P_f^l} - mh^{xl}_{T_i,P_i}$$

$$= m^{xl}h^{xl}_{T_f,P_f^{xl}} + m^l\left[h^{xl}_{T^l,P_f^{xl}} + \int_{P_f^{xl}}^{P_f^l}\left(\frac{\partial h^{xl}}{\partial P}\right)dP + \Delta h_{fusion}\right] - mh^{xl}_{T_i,P_i} \tag{C3}$$

$$= m^{xl}h^{xl}_{T_f,P_f^{xl}} + mh^{xl}_{T_f,P_f^{xl}} + m^l(1 - \alpha T_f)v^{xl}(P_f^l - P_f^{xl})$$
$$+ m^l \Delta h_{fusion} - mh^{xl}_{T_i,P_i}, \tag{C4}$$

where $h^{xl}_{T_f,P_f^{xl}}$, α, v^{xl} and Δh_{fusion} represent the specific (per unit mass) enthalpy of the crystal or crystalline assembledge at T_f and P_f^{xl} (the final

pressure acting on the residual crystals[28]), the isobaric expansivity, the specific volume of the crystal (assumed incompressible) and the specific heat of fusion. Note that Δh_{fusion} depends on P and T; the values used here correspond to the average Δh_{fusion} along the PT path of the rising diapir. Thermodynamic data for evaluating the dependence of Δh_{fusion} on P and T for a number of important silicate phases are tabulated in Carmichael et al., 1977.

Rearranging (C4), using the thermodynamic identity

$$(h^{xl}_{T_f,P_f^{xl}} - h^{xl}_{T_i,P_i}) = c^{xl}(T_f - T_i) + (1 - \alpha T_f)v^{xl}(P_f^{xl} - P_i) \tag{C5}$$

and noting $(P_f^{xl} - P_i) = \hat{\rho}g(Z_f^{xl} - Z_i)$ one finally can show

$$\frac{\Delta H}{m} = c^{xl}(T_f - T_i) + (1 - \alpha T_f)g(Z_f^{xl} - Z_i)$$
$$+ f\{(1 - \alpha T_f)g(Z_f^l - Z_f^{xl}) + \Delta h_{fusion}\}$$

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